AQA Maths Mechanics 2

Past Paper Pack

2006-2015

General Certificate of Education
January 2006
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM2A/W**
Unit Mechanics 2A

Monday 16 January 2006 9.00 am to 10.15 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

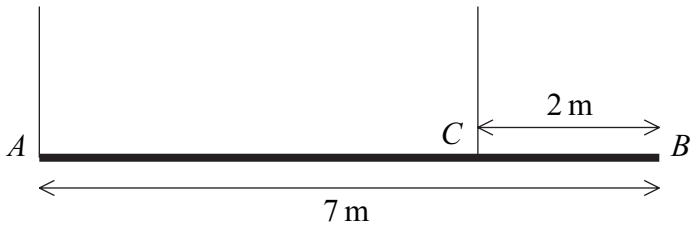
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2A/W.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Mechanics 2A has a **written paper and coursework**.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

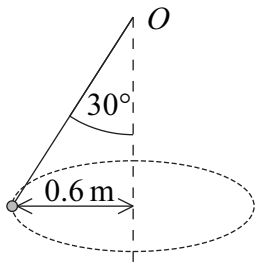
Answer **all** questions.

1 A uniform beam, AB, has mass 20 kg and length 7 metres. A rope is attached to the beam at A. A second rope is attached to the beam at the point C, which is 2 metres from B. Both of the ropes are vertical. The beam is in equilibrium in a horizontal position, as shown in the diagram.



Find the tensions in the two ropes. *(6 marks)*

2 A particle, of mass 2 kg, is attached to one end of a light inextensible string. The other end is fixed to the point O. The particle is set into motion, so that it describes a horizontal circle of radius 0.6 metres, with the string at an angle of 30° to the vertical. The centre of the circle is vertically below O.



(a) Show that the tension in the string is 22.6 N, correct to three significant figures. *(3 marks)*

(b) Find the speed of the particle. *(4 marks)*

3 A particle moves in a straight line and at time t has velocity v, where

$$v = 2t - 12e^{-t}, \quad t \geqslant 0$$

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) State the range of values of the acceleration of the particle. *(3 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

4 A car has a maximum speed of $42\,\text{m s}^{-1}$ when it is moving on a horizontal road. When the speed of the car is $v\,\text{m s}^{-1}$, it experiences a resistance force of magnitude $30v$ newtons.

(a) Show that the maximum power of the car is 52 920 W. *(2 marks)*

(b) The car has mass 1200 kg. It travels, from rest, up a slope inclined at 5° to the horizontal.

(i) Show that, when the car is travelling at its maximum speed $V\,\text{m s}^{-1}$ up the slope,

$$V^2 + 392 \sin 5° V - 1764 = 0$$ *(4 marks)*

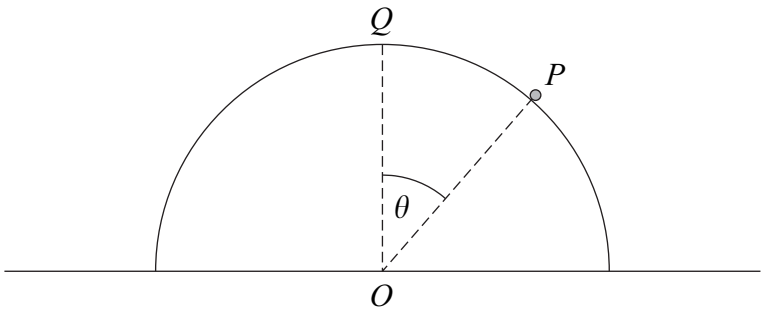
(ii) Hence find V. *(2 marks)*

5 A car, of mass 1600 kg, is travelling along a straight horizontal road at a speed of $20\,\text{m s}^{-1}$ when the driving force is removed. The car then freewheels and experiences a resistance force. The resistance force has magnitude $40v$ newtons, where $v\,\text{m s}^{-1}$ is the speed of the car after it has been freewheeling for t seconds.

Find an expression for v in terms of t. *(7 marks)*

Turn over for the next question

6 A particle P, of mass m kg, is placed at the point Q on the top of a smooth upturned hemisphere of radius 3 metres and centre O. The plane face of the hemisphere is fixed to a horizontal table. The particle is set into motion with an initial horizontal velocity of $2\,\mathrm{m\,s^{-1}}$. When the particle is on the surface of the hemisphere, the angle between OP and OQ is θ and the particle has speed $v\,\mathrm{m\,s^{-1}}$.



(a) Show that $v^2 = 4 + 6g(1 - \cos\theta)$. *(4 marks)*

(b) Find the value of θ when the particle leaves the hemisphere. *(5 marks)*

7 A particle, of mass 10 kg, is attached to one end of a light elastic string of natural length 0.4 metres and modulus of elasticity 100 N. The other end of the string is fixed to the point O.

(a) Find the length of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(b) The particle is pulled down and held at a point P, which is 1 metre vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 45 J. *(2 marks)*

(c) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\mathrm{m\,s^{-1}}$ when it is x metres **below *O***.

(i) Show that, while the string is taut,

$$v^2 = 39.6x - 25x^2 - 14.6$$ *(7 marks)*

(ii) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(3 marks)*

END OF QUESTIONS

General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MM2A/W**
Unit Mechanics 2A

Tuesday 6 June 2006 1.30 pm to 2.45 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2A/W.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Mechanics 2A has a **written paper and coursework**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves in a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively. At time t seconds, its position vector, **r** metres, is given by

$$\mathbf{r} = (2t^3 - t^2 + 6)\mathbf{i} + (8 - 4t^3 + t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

(b) (i) Find the velocity of the particle when $t = \frac{1}{3}$. *(2 marks)*

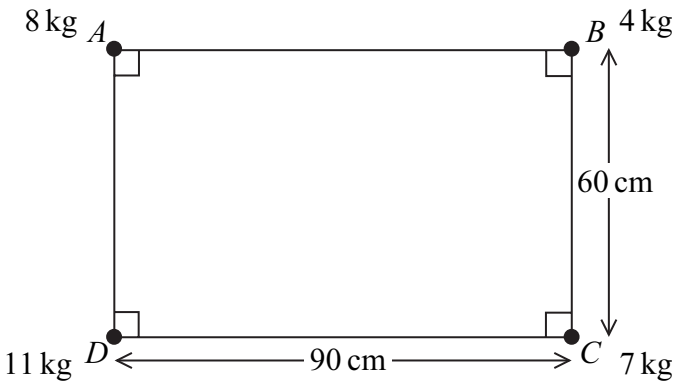
(ii) State the direction in which the particle is travelling at this time. *(1 mark)*

(c) Find the acceleration of the particle when $t = 4$. *(3 marks)*

(d) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 4$. *(3 marks)*

2 Particles of masses 8 kg, 4 kg, 7 kg and 11 kg are attached to the vertices A, B, C and D respectively of a light, rigid, rectangular framework $ABCD$.

In the framework, $CD = 90$ cm and $BC = 60$ cm, as shown in the diagram.



(a) Show that the centre of mass of the system of particles is 33 cm from AD. *(3 marks)*

(b) Find the distance of the centre of mass of the system of particles from AB. *(3 marks)*

(c) The framework is freely suspended from the corner A and hangs in equilibrium. Find the angle, in degrees, between the downward vertical and the side AB. *(3 marks)*

3 A block of mass 2 kg is placed on a horizontal surface. An elastic string has natural length 0.5 metres and modulus of elasticity 30 newtons. One end of the string is fixed to the surface at the point O and the other end is attached to the block. The block is pulled along the surface away from O until it is at the point P, where the length of OP is 1.8 metres. The block can be modelled as a particle.

(a) Calculate the elastic potential energy in the string when the block is at P. *(2 marks)*

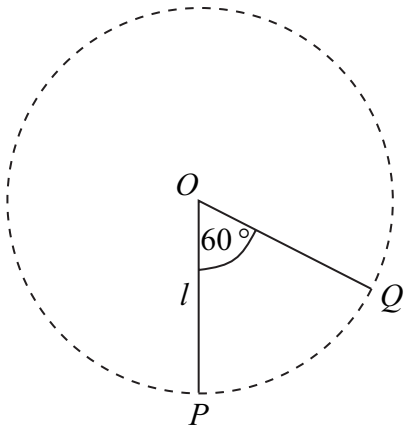
(b) Assume that the horizontal surface is smooth.

(i) The block is then released from P and moves towards O. Show that, when the block has moved 0.5 metres, its speed is $5.61\ \mathrm{m\,s^{-1}}$, correct to three significant figures. *(5 marks)*

(ii) Find the speed of the block when it reaches O. *(3 marks)*

(c) Assume that the horizontal surface is rough and that the coefficient of friction between the surface and the block is 0.1. Find the speed of the block when it reaches O. *(5 marks)*

4 A particle of mass m is suspended from a fixed point O by a light inextensible string of length l. The particle hangs in equilibrium at the point P vertically below O. The particle is then set into motion with a horizontal velocity U so that it moves in a complete vertical circle with centre O. The point Q on the circle is such that $\angle POQ = 60°$, as shown in the diagram.



(a) Find, in terms of g, l and U, the speed of the particle at Q. *(4 marks)*

(b) Find, in terms of g, l, m and U, the tension in the string when the particle is at Q. *(5 marks)*

Turn over for the next question

5 A car of mass 1200 kg travels round a roundabout on a horizontal, circular path at a constant speed of $14\,\mathrm{m\,s^{-1}}$. The radius of the circle is 50 metres. Assume that there is no resistance to the motion of the car and that the car can be modelled as a particle.

(a) A friction force, directed towards the centre of the roundabout, acts on the car as it moves. Show that the magnitude of this friction force is 4704 N. *(4 marks)*

(b) The coefficient of friction between the car and the road is μ. Show that $\mu \geqslant 0.4$. *(3 marks)*

6 A particle of mass 20 kg moves along a straight horizontal line. At time t seconds the velocity of the particle is $v\,\mathrm{m\,s^{-1}}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is $25\,\mathrm{m\,s^{-1}}$.

(a) Show that, at time t

$$v = \left(\frac{20 - t}{4}\right)^2$$

(7 marks)

(b) State the value of t when the particle comes to rest. *(1 mark)*

END OF QUESTIONS

General Certificate of Education
January 2007
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM2B**
Unit Mechanics 2B

Tuesday 16 January 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

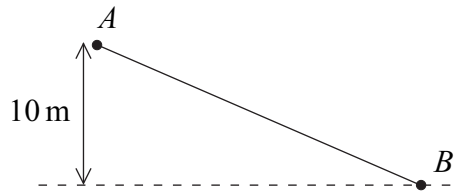
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A child, of mass 35 kg, slides down a slide in a water park. The child, starting from rest, slides from the point A to the point B, which is 10 metres vertically below the level of A, as shown in the diagram.



(a) In a simple model, all resistance forces are ignored.

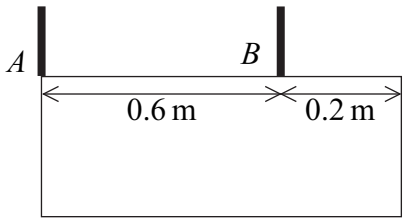
Use an energy method to find the speed of the child at B. *(3 marks)*

(b) State one resistance force that has been ignored in answering part (a). *(1 mark)*

(c) In fact, when the child slides down the slide, she reaches B with a speed of $12\,\text{m}\,\text{s}^{-1}$.

Given that the slide is 20 metres long and the sum of the resistance forces has a constant magnitude of F newtons, use an energy method to find the value of F. *(4 marks)*

2 A hotel sign consists of a uniform rectangular lamina of weight W. The sign is suspended in equilibrium in a vertical plane by two vertical light chains attached to the sign at the points A and B, as shown in the diagram. The edge containing A and B is horizontal.



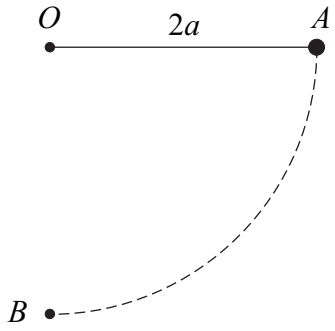
The tensions in the chains attached at A and B are T_A and T_B respectively.

(a) Draw a diagram to show the forces acting on the sign. *(1 mark)*

(b) Find T_A and T_B in terms of W. *(4 marks)*

(c) Explain how you have used the fact that the lamina is uniform in answering part (b). *(1 mark)*

3 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B, which is directly below O. The points O, A and B are as shown in the diagram.

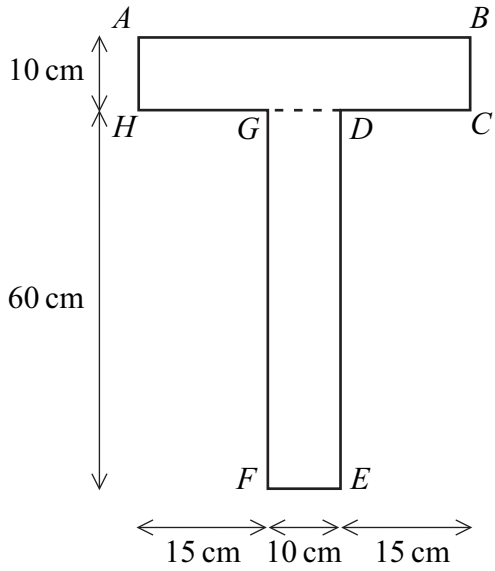


(a) Show that the speed of the particle at B is $2\sqrt{ag}$. *(3 marks)*

(b) Find the tension in the string as the particle passes through B. Give your answer in terms of m and g. *(3 marks)*

Turn over for the next question

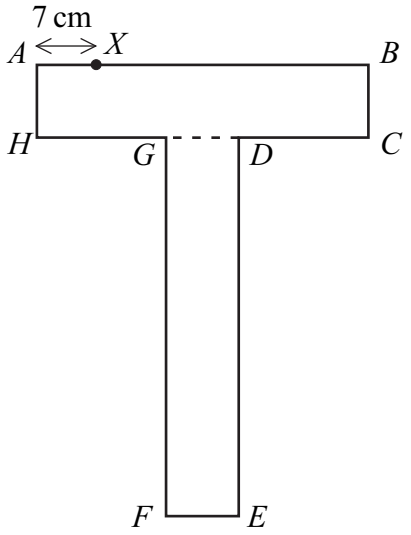
4 A uniform T-shaped lamina is formed by rigidly joining two rectangles *ABCH* and *DEFG*, as shown in the diagram.



(a) Show that the centre of mass of the lamina is 26 cm from the edge *AB*. *(4 marks)*

(b) Explain why the centre of mass of the lamina is 5 cm from the edge *GF*. *(1 mark)*

(c) The point *X* is on the edge *AB* and is 7 cm from *A*, as shown in the diagram below.



The lamina is freely suspended from *X* and hangs in equilibrium.

Find the angle between the edge *AB* and the vertical, giving your answer to the nearest degree. *(4 marks)*

5 Tom is on a fairground ride.

Tom's position vector, **r** metres, at time t seconds is given by

$$\mathbf{r} = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + (10 - 0.4t)\mathbf{k}$$

The perpendicular unit vectors **i** and **j** are in the horizontal plane and the unit vector **k** is directed vertically upwards.

(a) (i) Find Tom's position vector when $t = 0$. *(1 mark)*

(ii) Find Tom's position vector when $t = 2\pi$. *(1 mark)*

(iii) Write down the first **two** values of t for which Tom is directly below his starting point. *(2 marks)*

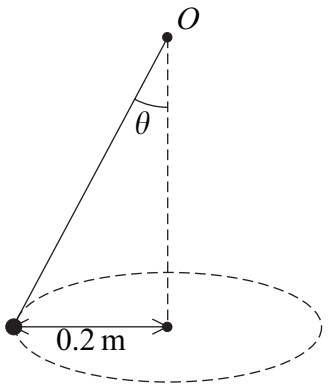
(b) Find an expression for Tom's velocity at time t. *(3 marks)*

(c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude. State the magnitude of the resultant force. *(5 marks)*

Turn over for the next question

6 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O. The angle between the string and the vertical is θ, as shown in the diagram.



(a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) The radius of the circle is 0.2 metres.

Find, in terms of π, the magnitude of the acceleration of the particle. *(2 marks)*

(c) The mass of the particle is m kg and the tension in the string is T newtons.

(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(ii) Explain why $T\cos\theta = mg$. *(1 mark)*

(iii) Find the value of θ, giving your answer to the nearest degree. *(5 marks)*

7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of $V\,\text{m s}^{-1}$, they experience a total resistance force of magnitude kV newtons, where k is a constant.

(a) The maximum speed of the motorcycle and its rider is $60\,\text{m s}^{-1}$.

Show that $k = 20$. *(3 marks)*

(b) When the motorcycle is travelling at $20\,\text{m s}^{-1}$, the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for t seconds, its speed is $v\,\text{m s}^{-1}$ and the magnitude of the resistance force is $20v$ newtons.

The mass of the motorcycle and its rider is 500 kg.

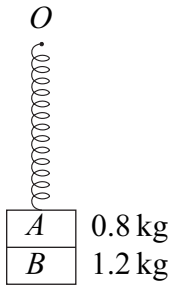
(i) Show that $\dfrac{\text{d}v}{\text{d}t} = -\dfrac{v}{25}$. *(2 marks)*

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from $20\,\text{m s}^{-1}$ to $10\,\text{m s}^{-1}$. *(6 marks)*

Turn over for the next question

8 Two small blocks, A and B, of masses 0.8 kg and 1.2 kg respectively, are stuck together. A spring has natural length 0.5 metres and modulus of elasticity 49 N. One end of the spring is attached to the top of the block A and the other end of the spring is attached to a fixed point O.

(a) The system hangs in equilibrium with the blocks stuck together, as shown in the diagram.



Find the extension of the spring. *(3 marks)*

(b) Show that the elastic potential energy of the spring when the system is in equilibrium is 1.96 J. *(2 marks)*

(c) The system is hanging in this equilibrium position when block B falls off and block A begins to move vertically upwards.

Block A next comes to rest when the spring is **compressed** by x metres.

(i) Show that x satisfies the equation

$$x^2 + 0.16x - 0.008 = 0$$ *(5 marks)*

(ii) Find the value of x. *(2 marks)*

END OF QUESTIONS

General Certificate of Education
June 2007
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM2B**
Unit Mechanics 2B

Thursday 7 June 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A hot air balloon moves vertically upwards with a constant velocity. When the balloon is at a height of 30 metres above ground level, a box of mass 5 kg is released from the balloon.

After the box is released, it initially moves vertically upwards with speed $10\,\mathrm{m\,s^{-1}}$.

(a) Find the initial kinetic energy of the box. *(2 marks)*

(b) Show that the kinetic energy of the box when it hits the ground is 1720 J. *(3 marks)*

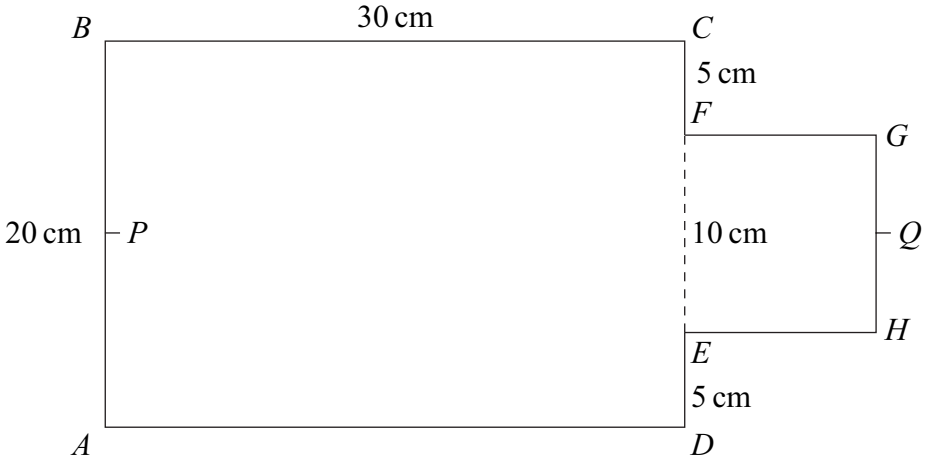
(c) Hence find the speed of the box when it hits the ground. *(3 marks)*

(d) State **two** modelling assumptions which you have made. *(2 marks)*

2 A uniform lamina is in the shape of a rectangle $ABCD$ and a square $EFGH$, as shown in the diagram.

The length AB is 20 cm, the length BC is 30 cm, the length DE is 5 cm and the length EF is 10 cm.

The point P is the midpoint of AB and the point Q is the midpoint of HG.



(a) Explain why the centre of mass of the lamina lies on PQ. *(1 mark)*

(b) Find the distance of the centre of mass of the lamina from AB. *(4 marks)*

(c) The lamina is freely suspended from A.

Find, to the nearest degree, the angle between AD and the vertical when the lamina is in equilibrium. *(4 marks)*

3 A particle has mass 800 kg. A single force of $(2400\,\mathbf{i} - 4800t\,\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) At time $t = 0$, the velocity of the particle is $(6\mathbf{i} + 30\mathbf{j})\,\mathrm{m\,s^{-1}}$. The velocity of the particle at time t is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

Show that

$$\mathbf{v} = (6 + 3t)\,\mathbf{i} + (30 - 3t^2)\,\mathbf{j}$$ *(4 marks)*

(c) Initially, the particle is at the point with position vector $(2\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

4 A uniform plank is 10 m long and has mass 15 kg. It is placed on horizontal ground at the edge of a vertical river bank, so that 2 m of the plank is projecting over the edge, as shown in the diagram below.

(a) A woman of mass 50 kg stands on the part of the plank which projects over the river.

Find the greatest distance from the river bank at which she can safely stand. *(3 marks)*

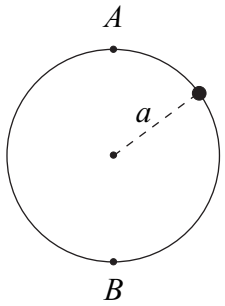
(b) The woman wishes to stand safely at the end of the plank which projects over the river.

Find the minimum mass which she should place on the other end of the plank so that she can do this. *(4 marks)*

(c) State how you have used the fact that the plank is uniform in your solution. *(1 mark)*

(d) State one other modelling assumption which you have made. *(1 mark)*

5 A bead of mass m moves on a smooth circular ring of radius a which is fixed in a vertical plane, as shown in the diagram. Its speed at A, the highest point of its path, is v and its speed at B, the lowest point of its path, is $7v$.



(a) Show that $v = \sqrt{\frac{ag}{12}}$. *(5 marks)*

(b) Find the reaction of the ring on the bead, in terms of m and g, when the bead is at A. *(4 marks)*

6 An elastic string has one end attached to a point O, fixed on a horizontal table. The other end of the string is attached to a particle of mass 5 kilograms. The elastic string has natural length 2 metres and modulus of elasticity 200 newtons. The particle is pulled so that it is 2.5 metres from the point O and it is then released from rest on the table.

(a) Calculate the elastic potential energy when the particle is 2.5 m from the point O. *(2 marks)*

(b) If the table is smooth, show that the speed of the particle when the string becomes slack is $\sqrt{5}$ m s^{-1}. *(3 marks)*

(c) The table is, in fact, rough and the coefficient of friction between the particle and the table is 0.4 .

Find the speed of the particle when the string becomes slack. *(7 marks)*

7 A stone of mass m is moving along the smooth horizontal floor of a tank which is filled with a viscous liquid. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude λmv, where λ is a constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v \qquad \textit{(2 marks)}$$

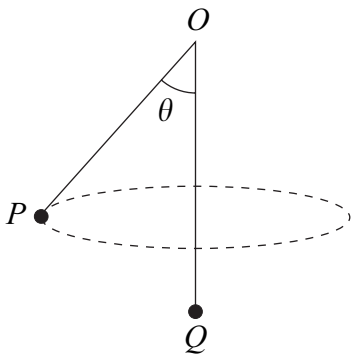
(b) The initial speed of the stone is U.

Show that

$$v = U\mathrm{e}^{-\lambda t} \qquad \textit{(4 marks)}$$

8 A particle, P, of mass 3 kg is attached to one end of a light inextensible string. The string passes through a smooth fixed ring, O, and a second particle, Q, of mass 5 kg is attached to the other end of the string. The particle Q hangs at rest vertically below the ring and the particle P moves with speed $4\,\mathrm{m\,s^{-1}}$ in a horizontal circle, as shown in the diagram.

The angle between OP and the vertical is θ.



(a) Explain why the tension in the string is 49 N. *(2 marks)*

(b) Find θ. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

END OF QUESTIONS

General Certificate of Education
January 2008
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM2B**
Unit Mechanics 2B

Tuesday 15 January 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A ball is thrown vertically upwards from ground level with an initial speed of $15\,\text{m}\,\text{s}^{-1}$. The ball has a mass of 0.6 kg. Assume that the only force acting on the ball after it is thrown is its weight.

(a) Calculate the initial kinetic energy of the ball. *(2 marks)*

(b) By using conservation of energy, find the maximum height above ground level reached by the ball. *(3 marks)*

(c) By using conservation of energy, find the kinetic energy and the speed of the ball when it is at a height of 3 m above ground level. *(4 marks)*

(d) State one modelling assumption which has been made. *(1 mark)*

2 A particle moves in a straight line and at time t it has velocity v, where

$$v = 3t^2 - 2\sin 3t + 6$$

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

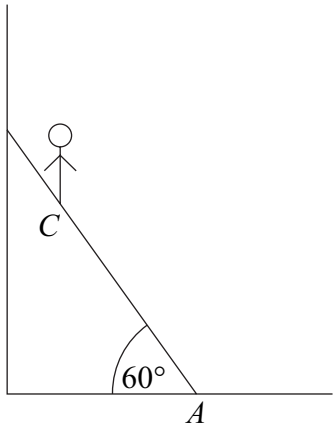
(ii) When $t = \frac{\pi}{3}$, show that the acceleration of the particle is $2\pi + 6$. *(2 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

3 A uniform ladder of length 4 metres and mass 20 kg rests in equilibrium with its foot, A, on a rough horizontal floor and its top leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall and the angle between the ladder and the floor is 60°.

A man of mass 80 kg is standing at point C on the ladder. With the man in this position, the ladder is on the point of slipping. The coefficient of friction between the ladder and the floor is 0.4. The man may be modelled as a particle at C.



(a) Draw a diagram to show the forces acting on the ladder. *(2 marks)*

(b) Show that the magnitude of the frictional force between the ladder and the ground is 392 N. *(3 marks)*

(c) Find the distance AC. *(6 marks)*

4 A particle moves in a horizontal plane under the action of a single force, **F** newtons. The unit vectors **i** and **j** are directed east and north respectively. At time t seconds, the position vector, **r** metres, of the particle is given by

$$\mathbf{r} = (t^3 - 3t^2 + 4)\mathbf{i} + (4t + t^2)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

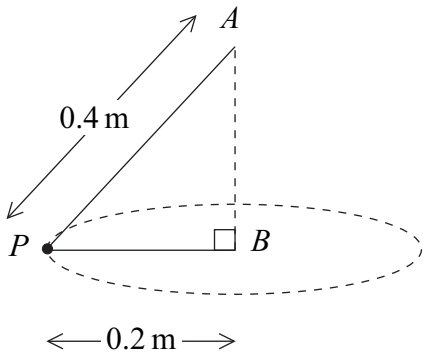
(b) The mass of the particle is 3 kg.

(i) Find an expression for **F** at time t. *(3 marks)*

(ii) Find the magnitude of **F** when $t = 3$. *(2 marks)*

(c) Find the value of t when **F** acts due north. *(2 marks)*

5 Two light inextensible strings, of lengths 0.4 m and 0.2 m, each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the points A and B respectively. The point A is vertically above the point B. The particle moves in a horizontal circle, centre B and radius 0.2 m, at a speed of $2\,\text{m}\,\text{s}^{-1}$. The particle and strings are shown in the diagram.



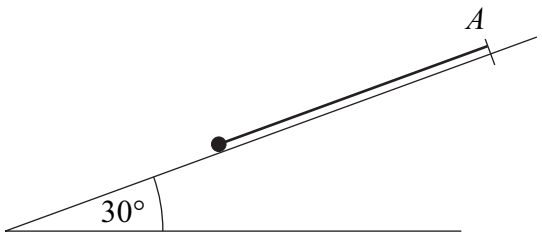
(a) Calculate the magnitude of the acceleration of the particle. *(2 marks)*

(b) Show that the tension in string PA is 45.3 N, correct to three significant figures. *(4 marks)*

(c) Find the tension in string PB. *(3 marks)*

6 A light elastic string has one end attached to a point A fixed on a smooth plane inclined at 30° to the horizontal. The other end of the string is attached to a particle of mass 6 kg. The elastic string has natural length 4 metres and modulus of elasticity 300 newtons.

The particle is pulled down the plane in the direction of the line of greatest slope through A. The particle is released from rest when it is 5.5 metres from A.

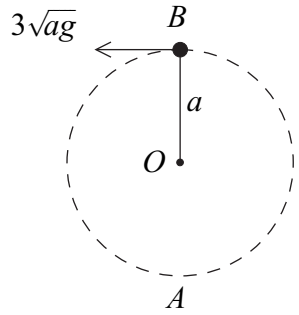


(a) Calculate the elastic potential energy of the string when the particle is 5.5 metres from the point A. *(2 marks)*

(b) Show that the speed of the particle when the string becomes slack is $3.66\,\text{m}\,\text{s}^{-1}$, correct to three significant figures. *(5 marks)*

(c) Show that the particle will **not** reach point A in the subsequent motion. *(3 marks)*

7 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end. The particle is moving in a vertical circle, centre O. When the particle is at B, vertically above O, the string is taut and the particle is moving with speed $3\sqrt{ag}$.



(a) Find, in terms of g and a, the speed of the particle at the lowest point, A, of its path. *(4 marks)*

(b) Find, in terms of g and m, the tension in the string when the particle is at A. *(4 marks)*

8 A car of mass 600 kg is driven along a straight horizontal road. The resistance to motion of the car is kv^2 newtons, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the car at time t seconds and k is a constant.

(a) When the engine of the car has power 8 kW, show that the equation of motion of the car is

$$600\frac{\mathrm{d}v}{\mathrm{d}t} - \frac{8000}{v} + kv^2 = 0 \qquad \textit{(4 marks)}$$

(b) When the velocity of the car is $20\,\mathrm{m\,s^{-1}}$, the engine is turned off.

(i) Show that the equation of motion of the car now becomes

$$600\frac{\mathrm{d}v}{\mathrm{d}t} = -kv^2 \qquad \textit{(1 mark)}$$

(ii) Find, in terms of k, the time taken for the velocity of the car to drop to $10\,\mathrm{m\,s^{-1}}$. *(5 marks)*

END OF QUESTIONS

General Certificate of Education
June 2008
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM2B**
Unit Mechanics 2B

Friday 6 June 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves in a straight line and at time t seconds has velocity $v\,\mathrm{m\,s^{-1}}$, where

$$v = 6t^2 + 4t - 7, \quad t \geqslant 0$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

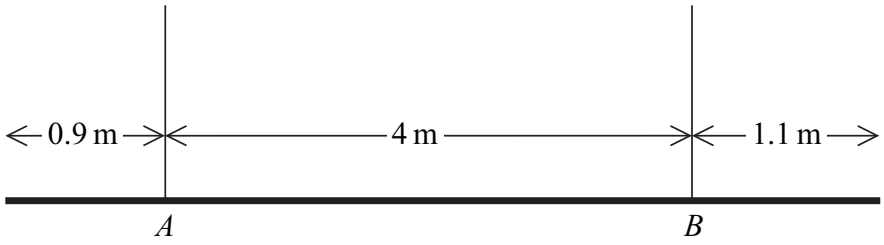
(b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. *(2 marks)*

(c) When $t = 0$, the displacement of the particle from the origin is 5 metres.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

2 A uniform plank, of length 6 metres, has mass 40 kg. The plank is held in equilibrium in a horizontal position by two vertical ropes attached to the plank at A and B, as shown in the diagram.



(a) Draw a diagram to show the forces acting on the plank. *(1 mark)*

(b) Show that the tension in the rope attached to the plank at B is $21g$ N. *(3 marks)*

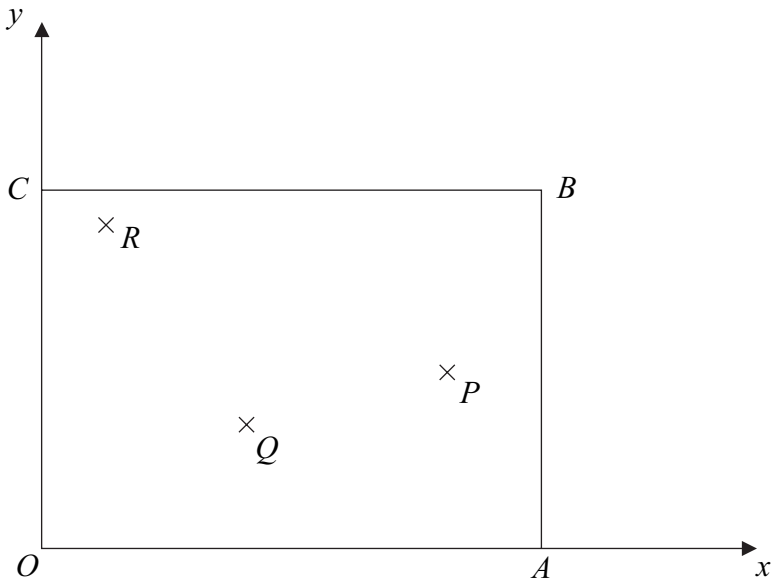
(c) Find the tension in the rope that is attached to the plank at A. *(2 marks)*

(d) State where in your solution you have used the fact that the plank is uniform. *(1 mark)*

3 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

Take OA and OC as the x- and y-axes, as shown.

Particle P has mass 1 kg and is attached at the point (25, 10).
Particle Q has mass 4 kg and is attached at the point (12, 7).
Particle R has mass 5 kg and is attached at the point (4, 18).



Find the coordinates of the centre of mass of the three particles. *(4 marks)*

4 A van, of mass 1500 kg, has a maximum speed of $50\,\text{m s}^{-1}$ on a straight horizontal road. When the van travels at a speed of $v\,\text{m s}^{-1}$, it experiences a resistance force of magnitude $40v$ newtons.

(a) Show that the maximum power of the van is 100 000 watts. *(2 marks)*

(b) The van is travelling along a straight horizontal road.

Find the maximum possible acceleration of the van when its speed is $25\,\text{m s}^{-1}$.
(3 marks)

(c) The van starts to climb a hill which is inclined at 6° to the horizontal. Find the maximum possible constant speed of the van as it travels in a straight line up the hill.
(6 marks)

5 A particle moves on a horizontal plane in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the particle's position vector, **r** metres, is given by

$$\mathbf{r} = 8\left(\cos\tfrac{1}{4}t\right)\mathbf{i} - 8\left(\sin\tfrac{1}{4}t\right)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Show that the speed of the particle is a constant. *(3 marks)*

(c) Prove that the particle is moving in a circle. *(2 marks)*

(d) Find the angular speed of the particle. *(2 marks)*

(e) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(f) State the magnitude of the acceleration of the particle. *(1 mark)*

6 A car, of mass m, is moving along a straight smooth horizontal road. At time t, the car has speed v. As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

(a) Show that

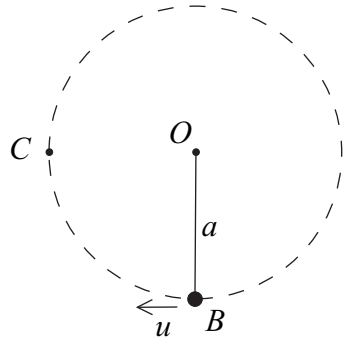
$$\frac{\mathrm{d}v}{\mathrm{d}t} = -0.05v \qquad \textit{(1 mark)}$$

(b) When $t = 0$, the speed of the car is $20\,\mathrm{m\,s^{-1}}$.

Show that $v = 20\mathrm{e}^{-0.05t}$. *(4 marks)*

(c) Find the time taken for the speed of the car to reduce to $10\,\mathrm{m\,s^{-1}}$. *(3 marks)*

7 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string, of length a. The bead is then set into circular motion with the string taut at B, where B is vertically below O, with a horizontal speed u.



(a) Given that the string does not become slack, show that the least value of u required for the bead to make complete revolutions about O is $\sqrt{5ag}$. *(5 marks)*

(b) In the case where $u = \sqrt{5ag}$, find, in terms of g and m, the tension in the string when the bead is at the point C, which is at the same horizontal level as O, as shown in the diagram. *(3 marks)*

(c) State one modelling assumption that you have made in your solution. *(1 mark)*

Turn over for the next question

8 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\frac{\lambda x}{l}$ when its extension is x.

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\frac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point O.

(i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(ii) The particle is pulled down and held at the point P, which is 0.9 metres vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 11.25 J. *(2 marks)*

(iii) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\mathrm{m\,s^{-1}}$ when it is x metres **above** ***P***.

Show that, while the string is taut,

$$v^2 = 10.4x - 50x^2$$ *(7 marks)*

(iv) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(2 marks)*

END OF QUESTIONS

General Certificate of Education
January 2009
Advanced Level Examination

MATHEMATICS MM2B
Unit Mechanics 2B

Thursday 29 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves along a straight line. At time t, it has velocity v, where

$$v = 4t^3 - 8\sin 2t + 5$$

When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

2 A stone, of mass 6 kg, is thrown vertically upwards with a speed of $12\,\text{m s}^{-1}$ from a point at a height of 4 metres above ground level.

(a) Calculate the initial kinetic energy of the stone. *(2 marks)*

(b) (i) Show that the kinetic energy of the stone when it hits the ground is 667 J, correct to three significant figures. *(2 marks)*

(ii) Hence find the speed of the stone when it hits the ground. *(3 marks)*

(iii) State two modelling assumptions that you have made. *(2 marks)*

3 A particle moves on a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the position vector of the particle is **r** metres, where

$$\mathbf{r} = \left(2e^{\frac{1}{2}t} - 8t + 5\right)\mathbf{i} + (t^2 - 6t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

(b) (i) Find the speed of the particle when $t = 3$. *(2 marks)*

(ii) State the direction in which the particle is travelling when $t = 3$. *(1 mark)*

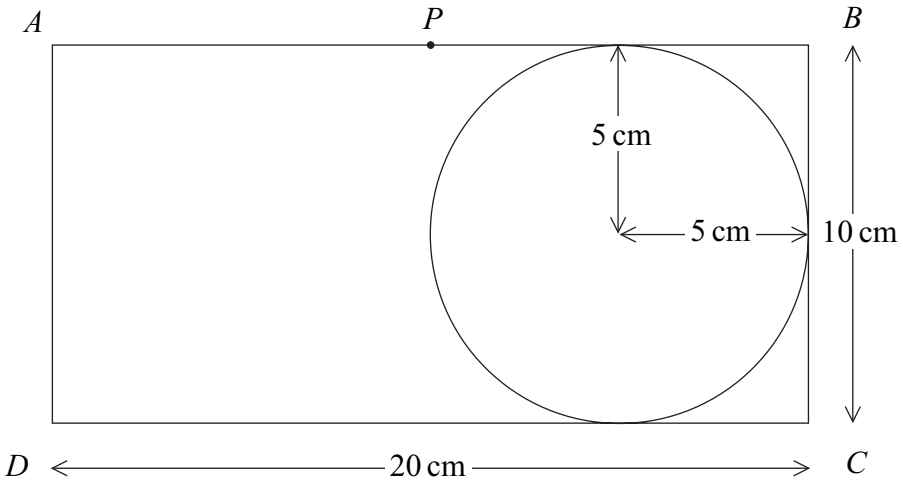
(c) Find the acceleration of the particle when $t = 3$. *(3 marks)*

(d) The mass of the particle is 7 kg.

Find the magnitude of the resultant force on the particle when $t = 3$. *(3 marks)*

4 A uniform rectangular lamina $ABCD$ has a mass of 8 kg. The side AB has length 20 cm, the side BC has length 10 cm, and P is the mid-point of AB.

A uniform circular lamina, of mass 2 kg and radius 5 cm, is fixed to the rectangular lamina to form a sign. The centre of the circular lamina is 5 cm from each of AB and BC, as shown in the diagram.



(a) Find the distance of the centre of mass of the sign from AD. *(3 marks)*

(b) Write down the distance of the centre of mass of the sign from AB. *(1 mark)*

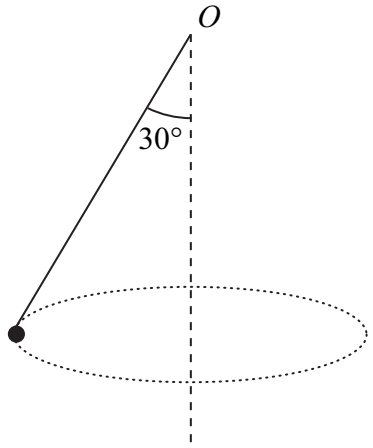
(c) The sign is freely suspended from P.

Find the angle between AD and the vertical when the sign is in equilibrium. *(4 marks)*

(d) Explain how you have used the fact that each lamina is uniform in your solution to this question. *(1 mark)*

Turn over for the next question

5 A particle, of mass 6 kg, is attached to one end of a light inextensible string. The other end of the string is attached to the fixed point O. The particle is set in motion, so that it moves in a horizontal circle at constant speed, with the string at an angle of 30° to the vertical. The centre of this circle is vertically below O.



The particle moves in a horizontal circle with an angular speed of 40 revolutions per minute.

(a) Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) Show that the tension in the string is 67.9 N, correct to three significant figures. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

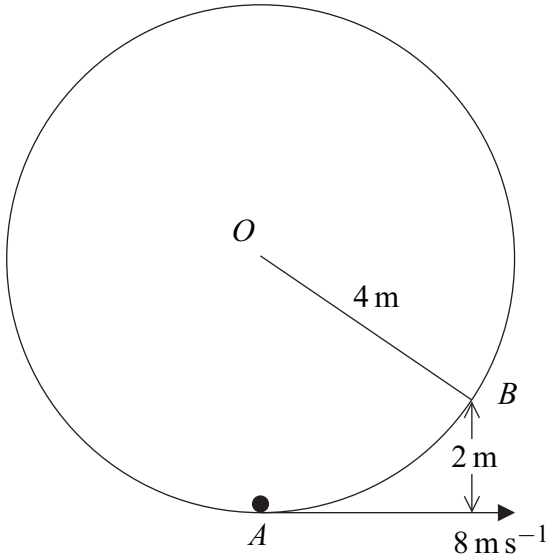
6 A train, of mass 60 tonnes, travels on a straight horizontal track. It has a maximum speed of $40\,\text{m}\,\text{s}^{-1}$ when its engine is working at 800 kW.

(a) Find the magnitude of the resistance force acting on the train when the train is travelling at its maximum speed. *(3 marks)*

(b) When the train is travelling at $40\,\text{m}\,\text{s}^{-1}$, the power is turned off. Assume that the resistance force is constant and is equal to that found in part (a). Also assume that this resistance force is the only horizontal force acting on the train.

Use an energy method to find how far the train travels when slowing from $40\,\text{m}\,\text{s}^{-1}$ to $36\,\text{m}\,\text{s}^{-1}$. *(4 marks)*

7 A hollow cylinder, of internal radius 4 m, is fixed so that its axis is horizontal. The point O is on this axis. A particle, of mass 6 kg, is set in motion so that it moves on the smooth **inner** surface of the cylinder in a vertical circle about O. Its speed at the point A, which is vertically below O, is $8\,\mathrm{m\,s^{-1}}$.



When the particle is at the point B, at a height of 2 m above A, find:

(a) its speed; *(3 marks)*

(b) the normal reaction between the cylinder and the particle. *(4 marks)*

8 A stone, of mass 0.05 kg, is moving along the smooth horizontal floor of a tank, which is filled with oil. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude $0.08v^2$.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -1.6v^2$$ *(2 marks)*

(b) The initial speed of the stone is $3\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \frac{15}{5 + 24t}$$ *(5 marks)*

Turn over ▶

9 A bungee jumper, of mass 80 kg, is attached to one end of a light elastic cord, of natural length 16 metres and modulus of elasticity 784 N. The other end of the cord is attached to a horizontal platform, which is at a height of 65 metres above the ground.

The bungee jumper steps off the platform at the point where the cord is attached and falls vertically. The bungee jumper can be modelled as a particle. Hooke's law can be assumed to apply throughout the motion and air resistance can be assumed to be negligible.

(a) Find the length of the cord when the acceleration of the bungee jumper is zero. *(3 marks)*

(b) The cord extends by x metres beyond its natural length before the bungee jumper first comes to rest.

(i) Show that $x^2 - 32x - 512 = 0$. *(4 marks)*

(ii) Find the distance above the ground at which the bungee jumper first comes to rest. *(4 marks)*

END OF QUESTIONS

General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS MM2B
Unit Mechanics 2B

Monday 15 June 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves under the action of a force, **F** newtons. At time t seconds, the velocity, $\mathbf{v}\,\mathrm{m\,s^{-1}}$, of the particle is given by

$$\mathbf{v} = (t^3 - 15t - 5)\mathbf{i} + (6t - t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(3 marks)*

(b) The mass of the particle is 4 kg.

(i) Show that, at time t,

$$\mathbf{F} = (12t^2 - 60)\mathbf{i} + (24 - 8t)\mathbf{j}$$ *(2 marks)*

(ii) Find the magnitude of **F** when $t = 2$. *(4 marks)*

2 A slide at a water park may be modelled as a smooth plane of length 20 metres inclined at 30° to the vertical. Anne, who has a mass of 55 kg, slides down the slide. At the top of the slide, she has an initial velocity of $3\,\mathrm{m\,s^{-1}}$ down the slide.

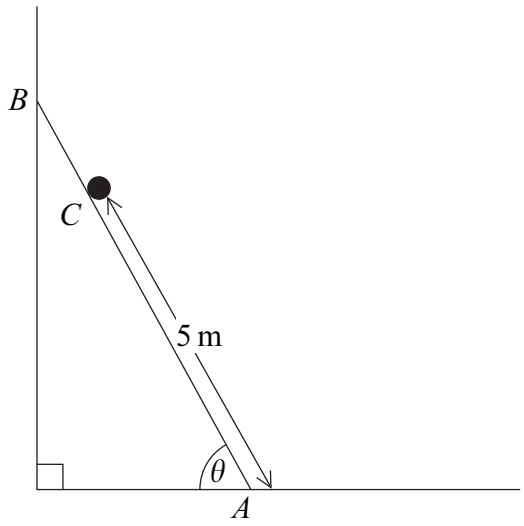
(a) Calculate Anne's initial kinetic energy. *(2 marks)*

(b) By using conservation of energy, find the kinetic energy and the speed of Anne after she has travelled the 20 metres. *(6 marks)*

(c) State one modelling assumption which you have made. *(1 mark)*

3 A uniform ladder, of length 6 metres and mass 22 kg, rests with its foot, A, on a rough horizontal floor and its top, B, leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall, and the angle between the ladder and the floor is θ.

A man, of mass 90 kg, is standing at point C on the ladder so that the distance AC is 5 metres. With the man in this position, the ladder is on the point of slipping. The coefficient of friction between the ladder and the horizontal floor is 0.6. The man may be modelled as a particle at C.

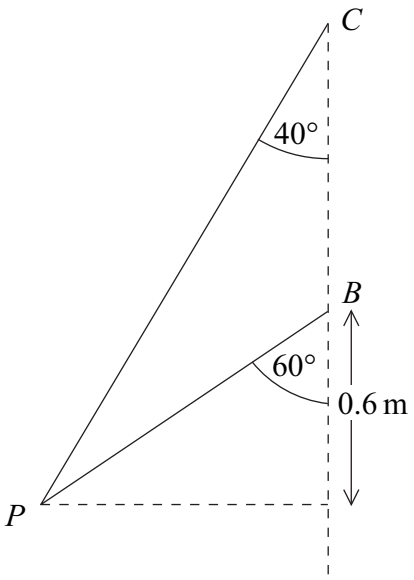


(a) Show that the magnitude of the frictional force between the ladder and the horizontal floor is 659 N, correct to three significant figures. *(4 marks)*

(b) Find the angle θ. *(5 marks)*

Turn over for the next question

4 Two light inextensible strings each have one end attached to a particle, P, of mass 6 kg. The other ends of the strings are attached to the fixed points B and C. The point C is vertically above the point B. The particle moves, at constant speed, in a horizontal circle, with centre 0.6 m below point B, with the strings inclined at 40° and 60° to the vertical, as shown in the diagram. Both strings are taut.



(a) As the particle moves in the horizontal circle, the tensions in the two strings are equal.

Show that the tension in the strings is 46.4 N, correct to three significant figures. *(4 marks)*

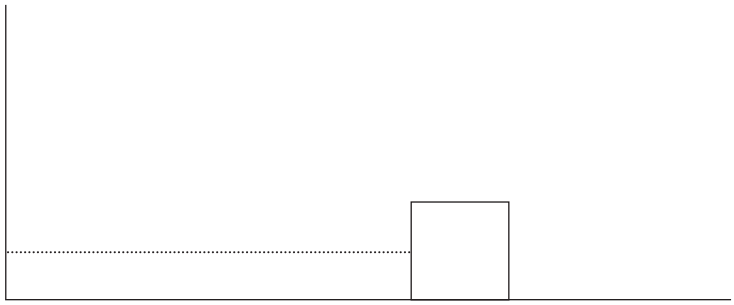
(b) Find the speed of the particle. *(4 marks)*

5 A train, of mass 600 tonnes, travels at constant speed up a slope inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{40}$. The speed of the train is $24\,\text{m}\,\text{s}^{-1}$ and it experiences total resistance forces of 200 000 N.

Find the power produced by the train, giving your answer in kilowatts. *(6 marks)*

6 A block, of mass 5 kg, is attached to one end of a length of elastic string. The other end of the string is fixed to a vertical wall. The block is placed on a horizontal surface.

The elastic string has natural length 1.2 m and modulus of elasticity 180 N. The block is pulled so that it is 2 m from the wall and is then released from rest. Whilst taut, the string remains horizontal. It may be assumed that, after the string becomes slack, it does not interfere with the movement of the block.



(a) Calculate the elastic potential energy when the block is 2 m from the wall. *(2 marks)*

(b) If the horizontal surface is smooth, find the speed of the block when it hits the wall. *(3 marks)*

(c) The surface is in fact rough and the coefficient of friction between the block and the surface is μ.

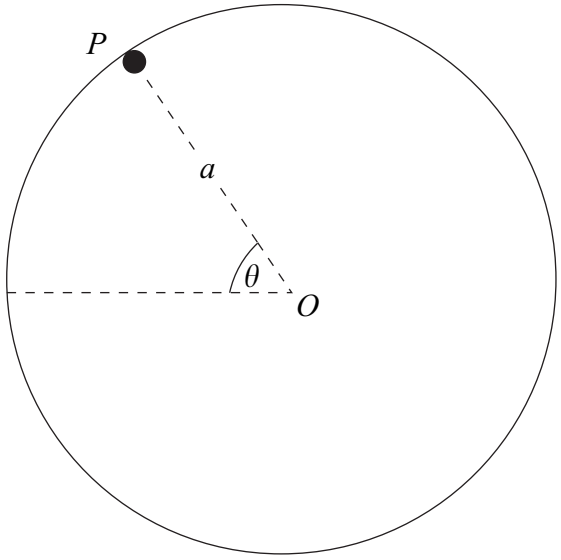
Find μ if the block comes to rest just as it reaches the wall. *(7 marks)*

Turn over for the next question

7 In crazy golf, a golf ball is hit so that it starts to move in a vertical circle on the inside of a smooth cylinder.

Model the golf ball as a particle, P, of mass m. The circular path of the golf ball has radius a and centre O. At time t, the angle between OP and the horizontal is θ, as shown in the diagram.

The golf ball has speed u at the lowest point of its circular path.



(a) Show that, while the golf ball is in contact with the cylinder, the reaction of the cylinder on the golf ball is

$$\frac{mu^2}{a} - 3mg\sin\theta - 2mg \qquad \textit{(6 marks)}$$

(b) Given that $u = \sqrt{3ag}$, the golf ball will not complete a vertical circle inside the cylinder. Find the angle which OP makes with the horizontal when the golf ball leaves the surface of the cylinder. *(4 marks)*

8 A stone, of mass m, is moving in a straight line along smooth horizontal ground.

At time t, the stone has speed v. As the stone moves, it experiences a total resistance force of magnitude $\lambda m v^{\frac{3}{2}}$, where λ is a constant. No other horizontal force acts on the stone.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v^{\frac{3}{2}}$$

(2 marks)

(b) The initial speed of the stone is $9\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \frac{36}{(2 + 3\lambda t)^2}$$

(7 marks)

(c) Find, in terms of λ, the time taken for the speed of the stone to drop to $4\,\mathrm{m\,s^{-1}}$.

(3 marks)

END OF QUESTIONS

General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MM2B

Unit Mechanics 2B

Wednesday 20 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

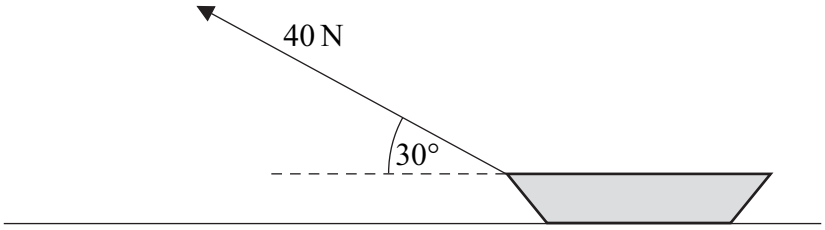
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

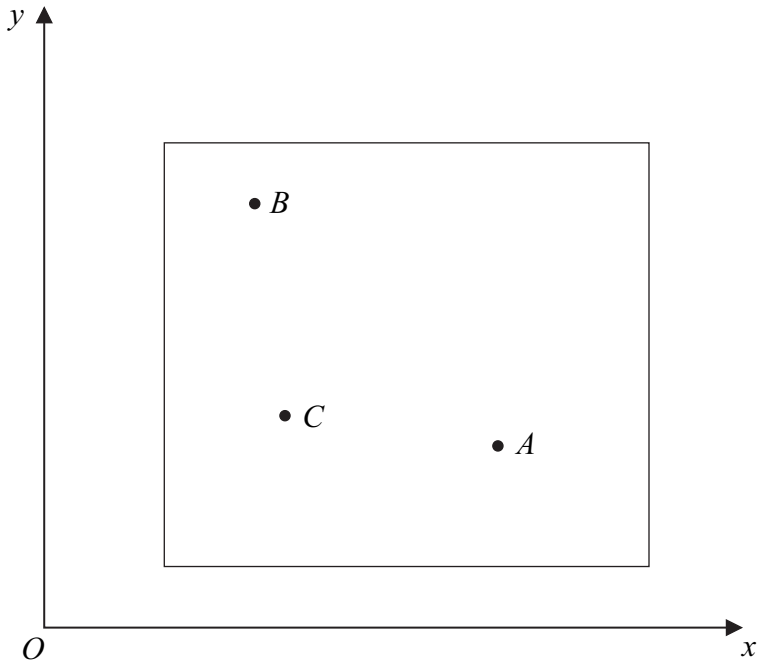
1 An inextensible rope is attached to a sledge which is at rest on a horizontal surface. A constant force of magnitude 40 newtons at an angle of 30° to the horizontal is applied to the sledge, as shown in the diagram.



Calculate the work done by the force as the sledge is moved 5 metres along the surface.

(3 marks)

2 A piece of modern art is modelled as a uniform lamina and three particles. The diagram shows the lamina, the three particles A, B and C, and the x- and y-axes.

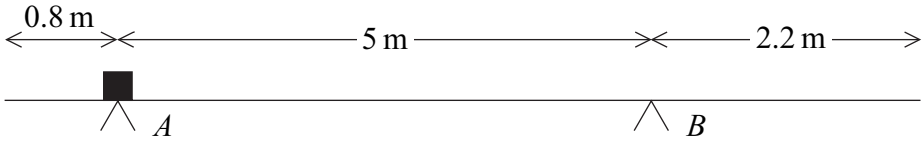


The lamina, which is fixed in the x–y plane, has mass 10 kg and its centre of mass is at the point $(12, 9)$.

The three particles are attached to the lamina.
Particle A has mass 3 kg and is at the point $(15, 6)$.
Particle B has mass 1 kg and is at the point $(7, 14)$.
Particle C has mass 6 kg and is at the point $(8, 7)$.

Find the coordinates of the centre of mass of the piece of modern art. *(6 marks)*

3 A uniform plank, of length 8 metres, has mass 30 kg. The plank is supported in equilibrium in a horizontal position by two smooth supports at the points A and B, as shown in the diagram. A block, of mass 20 kg, is placed on the plank at point A.



(a) Draw a diagram to show the forces acting on the plank. *(2 marks)*

(b) Show that the magnitude of the force exerted on the plank by the support at B is $19.2g$ newtons. *(3 marks)*

(c) Find the magnitude of the force exerted on the plank by the support at A. *(2 marks)*

(d) Explain how you have used the fact that the plank is uniform in your solution. *(1 mark)*

4 A particle moves so that at time t seconds its velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$ is given by

$$\mathbf{v} = (4t^3 - 12t + 3)\mathbf{i} + 5\mathbf{j} + 8t\mathbf{k}$$

(a) When $t = 0$, the position vector of the particle is $(-5\mathbf{i} + 6\mathbf{k})$ metres.

Find the position vector of the particle at time t. *(4 marks)*

(b) Find the acceleration of the particle at time t. *(2 marks)*

(c) Find the magnitude of the acceleration of the particle at time t. Do not simplify your answer. *(2 marks)*

(d) Hence find the time at which the magnitude of the acceleration is a minimum. *(2 marks)*

(e) The particle is moving under the action of a single variable force $\mathbf{F}$ newtons. The mass of the particle is 7 kg.

Find the minimum magnitude of $\mathbf{F}$. *(2 marks)*

5 A golf ball, of mass m kg, is moving in a straight line across smooth horizontal ground. At time t seconds, the golf ball has speed $v\,\mathrm{m\,s^{-1}}$. As the golf ball moves, it experiences a resistance force of magnitude $0.2mv^{\frac{1}{2}}$ newtons until it comes to rest. No other horizontal force acts on the golf ball.

Model the golf ball as a particle.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -0.2v^{\frac{1}{2}}$$ *(1 mark)*

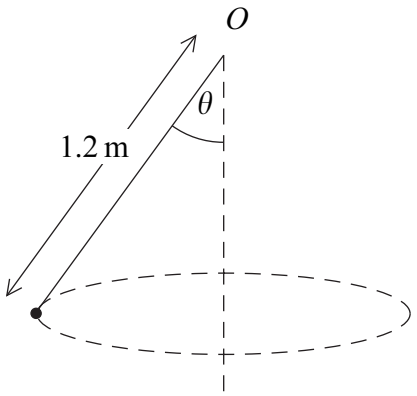
(b) When $t = 0$, the speed of the golf ball is $16\,\mathrm{m\,s^{-1}}$.

Show that $v = (4 - 0.1t)^2$. *(5 marks)*

(c) Find the value of t when $v = 1$. *(3 marks)*

(d) Find the distance travelled by the golf ball as its speed decreases from $16\,\mathrm{m\,s^{-1}}$ to $1\,\mathrm{m\,s^{-1}}$. *(4 marks)*

6 A particle, of mass 4 kg, is attached to one end of a light inextensible string of length 1.2 metres. The other end of the string is attached to a fixed point O. The particle moves in a horizontal circle at a constant speed. The angle between the string and the vertical is θ.



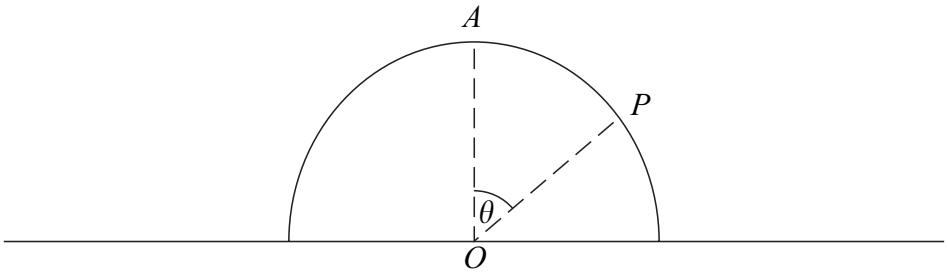
(a) Find the radius of the horizontal circle in terms of θ. *(1 mark)*

(b) The angular speed of the particle is 5 radians per second. Find θ. *(6 marks)*

7 A smooth hemisphere, of radius a and centre O, is fixed with its plane face on a horizontal surface. A particle, of mass m, can move freely on the surface of the hemisphere.

The particle is placed at the point A, the highest point of the hemisphere, and is set in motion along the surface with speed u.

(a) While the particle is in contact with the hemisphere at a point P, OP makes an angle θ with the upward vertical.



Show that the speed of the particle at P is

$$(u^2 + 2ga[1 - \cos\theta])^{\frac{1}{2}}$$ *(5 marks)*

(b) The particle leaves the surface of the hemisphere when $\theta = \alpha$.

Find $\cos\alpha$ in terms of a, u and g. *(5 marks)*

Turn over for the next question

8 A bungee jumper, of mass 49 kg, is attached to one end of a light elastic cord of natural length 22 metres and modulus of elasticity 1078 newtons. The other end of the cord is attached to a horizontal platform, which is at a height of 60 metres above the ground.

The bungee jumper steps off the platform at the point where the cord is attached, and falls vertically. The bungee jumper can be modelled as a particle. Assume that Hooke's Law applies whilst the cord is taut and that air resistance is negligible throughout the motion.

When the bungee jumper has fallen x metres, his speed is $v\,\mathrm{m\,s^{-1}}$.

(a) By considering energy, show that, when x is greater than 22,

$$5v^2 = 318x - 5x^2 - 2420$$ *(6 marks)*

(b) Explain why x must be greater than 22 for the equation in part (a) to be valid. *(1 mark)*

(c) Find the maximum value of x. *(4 marks)*

(d) (i) Show that the speed of the bungee jumper is a maximum when $x = 31.8$. *(3 marks)*

(ii) Hence find the maximum speed of the bungee jumper. *(2 marks)*

END OF QUESTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MM2B

Unit Mechanics 2B

Friday 18 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 A particle moves along a straight line through the origin. At time t, the displacement, s, of the particle from the origin is given by

$$s = 5t^2 + 3\cos 4t$$

Find the velocity of the particle at time t. *(3 marks)*

QUESTION PART REFERENCE

2 John is at the top of a cliff, looking out over the sea. He throws a rock, of mass 3 kg, horizontally with a velocity of $4\,\mathrm{m\,s^{-1}}$.

The rock falls a vertical distance of 51 metres to reach the surface of the sea.

(a) Calculate the kinetic energy of the rock when it is thrown. *(2 marks)*

(b) Calculate the potential energy lost by the rock when it reaches the surface of the sea. *(2 marks)*

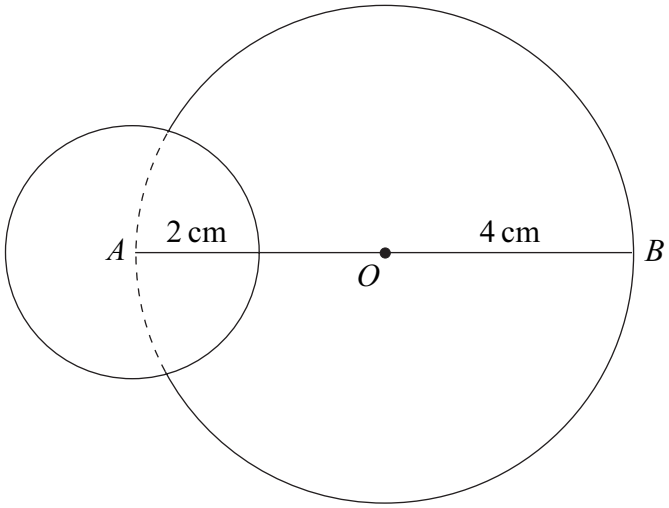
(c) (i) Find the kinetic energy of the rock when it reaches the surface of the sea.

(ii) Hence find the speed of the rock when it reaches the surface of the sea. *(4 marks)*

(d) State one modelling assumption which has been made. *(1 mark)*

QUESTION PART REFERENCE

3 A uniform circular lamina, of radius 4 cm and mass 0.4 kg, has a centre O, and AB is a diameter. To create a medal, a smaller uniform circular lamina, of radius 2 cm and mass 0.1 kg, is attached so that the centre of the smaller lamina is at the point A, as shown in the diagram.



(a) Explain why the centre of mass of the medal is on the line AB. *(1 mark)*

(b) Find the distance of the centre of mass of the medal from the point B. *(3 marks)*

QUESTION PART REFERENCE

4 A particle has mass 200 kg and moves on a smooth horizontal plane. A single horizontal force, $\left(400\cos\left(\frac{\pi}{2}t\right)\mathbf{i} + 600t^2\,\mathbf{j}\right)$ newtons, acts on the particle at time t seconds.

The unit vectors **i** and **j** are directed east and north respectively.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) When $t = 4$, the velocity of the particle is $(-3\mathbf{i} + 56\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find the velocity of the particle at time t. *(5 marks)*

(c) Find t when the particle is moving due west. *(3 marks)*

(d) Find the speed of the particle when it is moving due west. *(2 marks)*

QUESTION PART REFERENCE

5 A particle is moving along a straight line. At time t, the velocity of the particle is v. The acceleration of the particle throughout the motion is $-\frac{\lambda}{v^{\frac{1}{4}}}$, where λ is a positive constant. The velocity of the particle is u when $t = 0$.

Find v in terms of u, λ and t. *(7 marks)*

QUESTION PART REFERENCE

Do not write outside the box

6 When a car, of mass 1200 kg, travels at a speed of $v\,\mathrm{m\,s^{-1}}$, it experiences a resistance force of magnitude $30v$ newtons.

The car has a maximum constant speed of $48\,\mathrm{m\,s^{-1}}$ on a straight horizontal road.

(a) Show that the maximum power of the car is 69 120 watts. *(2 marks)*

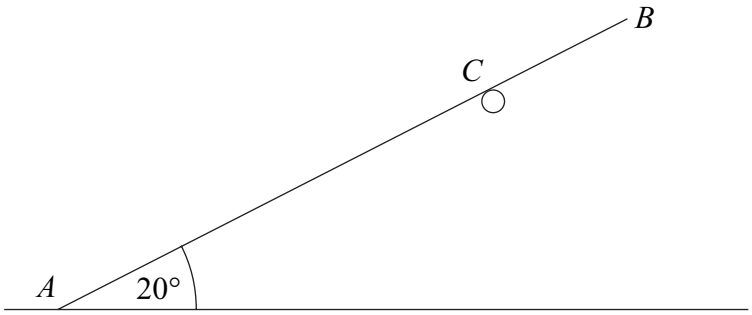
(b) The car is travelling along a straight horizontal road.

Find the maximum possible acceleration of the car when it is travelling at a speed of $40\,\mathrm{m\,s^{-1}}$. *(4 marks)*

(c) The car starts to descend a hill on a straight road which is inclined at an angle of 3° to the horizontal. Find the maximum possible constant speed of the car as it travels on this road down the hill. *(7 marks)*

QUESTION PART REFERENCE

7 A uniform rod AB, of length 4 m and mass 6 kg, rests in equilibrium with one end, A, on smooth horizontal ground. The rod rests on a rough horizontal peg at the point C, where AC is 3 m. The rod is inclined at an angle of 20° to the horizontal.



(a) Draw a diagram to show the forces acting on the rod. *(2 marks)*

(b) Find the magnitude of the normal reaction force between the rod and the ground. *(3 marks)*

(c) (i) Find the normal reaction acting on the rod at C.

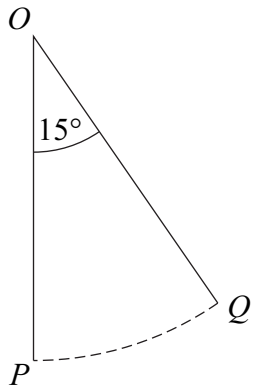
(ii) Find the friction force acting on the rod at C. *(5 marks)*

(d) In this position, the rod is on the point of slipping.

Calculate the coefficient of friction between the rod and the peg. *(2 marks)*

QUESTION PART REFERENCE

8 A particle is attached to one end of a light inextensible string of length 3 metres. The other end of the string is attached to a fixed point O. The particle is set into motion horizontally at point P with speed v, so that it describes part of a vertical circle whose centre is O. The point P is vertically below O.



The particle first comes momentarily to rest at the point Q, where OQ makes an angle of 15° to the vertical.

(a) Find the value of v. *(4 marks)*

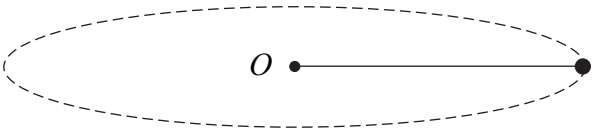
(b) When the particle is at rest at the point Q, the tension in the string is 22 newtons.

Find the mass of the particle. *(3 marks)*

QUESTION PART REFERENCE

9 A particle, of mass 8 kg, is attached to one end of a length of elastic string. The particle is placed on a smooth horizontal surface. The other end of the elastic string is attached to a point O fixed on the horizontal surface.

The elastic string has natural length 1.2 m and modulus of elasticity 192 N.



The particle is set in motion on the horizontal surface so that it moves in a circle, centre O, with constant speed $3\,\mathrm{m\,s^{-1}}$.

Find the radius of the circle. *(8 marks)*

QUESTION PART REFERENCE

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MM2B

Unit Mechanics 2B

Wednesday 26 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The velocity of a particle at time t seconds is $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (4 + 3t^2)\mathbf{i} + (12 - 8t)\mathbf{j}$$

(a) When $t = 0$, the particle is at the point with position vector $(5\mathbf{i} - 7\mathbf{j})\,\mathrm{m}$.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(4 marks)*

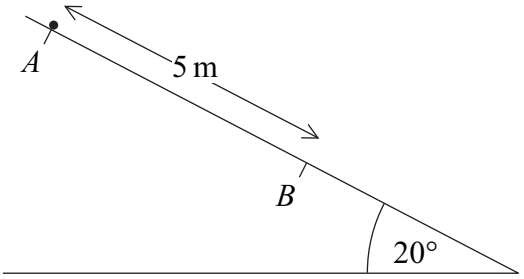
(b) Find the acceleration of the particle at time t. *(2 marks)*

(c) The particle has mass 2 kg.

Find the magnitude of the force acting on the particle when $t = 1$. *(4 marks)*

QUESTION PART REFERENCE

2 A particle is placed on a smooth plane which is inclined at an angle of 20° to the horizontal. The particle, of mass 4 kg, is released from rest at a point A and travels down the plane, passing through a point B. The distance AB is 5 m.



(a) Find the potential energy lost as the particle moves from point A to point B. *(2 marks)*

(b) Hence write down the kinetic energy of the particle when it reaches point B. *(1 mark)*

(c) Hence find the speed of the particle when it reaches point B. *(2 marks)*

QUESTION PART REFERENCE

3 A pump is being used to empty a flooded basement.

In one minute, 400 litres of water are pumped out of the basement.

The water is raised 8 metres and is ejected through a pipe at a speed of $2\,\mathrm{m\,s^{-1}}$.

The mass of 400 litres of water is 400 kg.

(a) Calculate the gain in potential energy of the 400 litres of water. *(1 mark)*

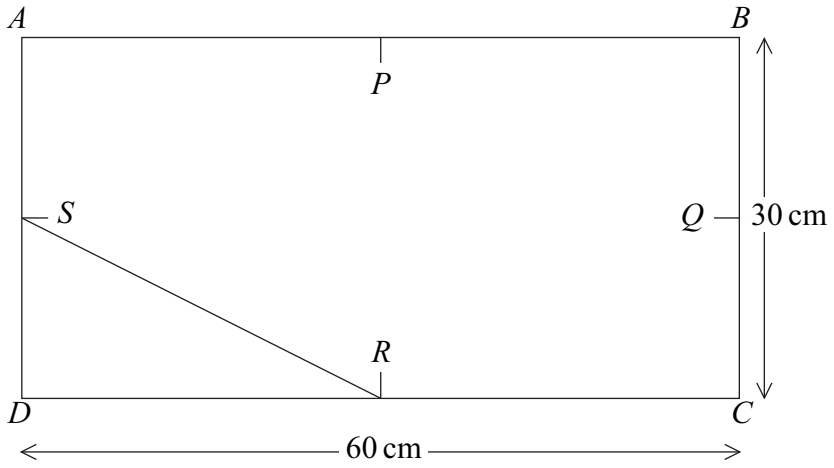
(b) Calculate the gain in kinetic energy of the 400 litres of water. *(1 mark)*

(c) Hence calculate the power of the pump, giving your answer in watts. *(2 marks)*

QUESTION PART REFERENCE

4 A uniform rectangular lamina $ABCD$ has a mass of 5 kg. The side AB has length 60 cm and the side BC has length 30 cm. The points P, Q, R and S are the mid-points of the sides, as shown in the diagram below.

A uniform triangular lamina SRD, of mass 4 kg, is fixed to the rectangular lamina to form a shop sign. The centre of mass of the triangular lamina SRD is 10 cm from the side AD and 5 cm from the side DC.



(a) Find the distance of the centre of mass of the shop sign from AD. *(3 marks)*

(b) Find the distance of the centre of mass of the shop sign from AB. *(3 marks)*

(c) The shop sign is freely suspended from P.

Find the angle between AB and the horizontal when the shop sign is in equilibrium. *(4 marks)*

(d) To ensure that the side AB is horizontal when the shop sign is freely suspended from point P, a particle of mass m kg is attached to the shop sign at point B.

Calculate m. *(3 marks)*

(e) Explain how you have used the fact that the rectangular lamina $ABCD$ is uniform in your solution to this question. *(1 mark)*

QUESTION PART REFERENCE

.....

.....

.....

.....

.....

5 (a) A shiny coin is on a rough horizontal turntable at a distance 0.8 m from its centre. The turntable rotates at a constant angular speed. The coefficient of friction between the shiny coin and the turntable is 0.3 .

Find the maximum angular speed, in radians per second, at which the turntable can rotate if the shiny coin is not going to slide. *(4 marks)*

(b) The turntable is stopped and the shiny coin is removed. An old coin is placed on the turntable at a distance 0.15 m from its centre. The turntable is made to rotate at a constant angular speed of 45 revolutions per minute.

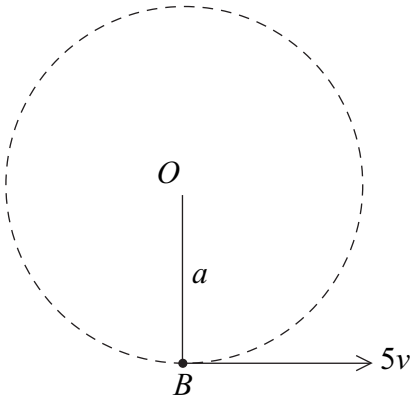
(i) Find the angular speed of the turntable in radians per second. *(2 marks)*

(ii) The old coin remains in the same position on the turntable.

Find the least value of the coefficient of friction between the old coin and the turntable needed to prevent the old coin from sliding. *(4 marks)*

QUESTION PART REFERENCE

6 A light inextensible string, of length a, has one end attached to a fixed point O. A small bead, of mass m, is attached to the other end of the string. The bead is moving in a vertical circle, centre O. When the bead is at B, vertically below O, the string is taut and the bead is moving with speed $5v$.



(a) The speed of the bead at the highest point of its path is $3v$.

Find v in terms of a and g. *(4 marks)*

(b) Find the ratio of the greatest tension to the least tension in the string, as the bead travels around its circular path. *(5 marks)*

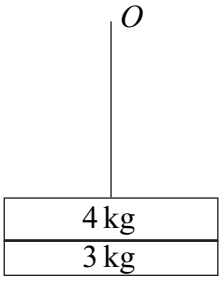
QUESTION PART REFERENCE

7 (a) An elastic string has natural length l and modulus of elasticity λ. The string is stretched from length l to length $l + e$.

Show, by integration, that the work done in stretching the string is $\frac{\lambda e^2}{2l}$. *(3 marks)*

(b) A block, of mass 4 kg, is attached to one end of a light elastic string. The string has natural length 2 m and modulus of elasticity 196 N. The other end of the string is attached to a fixed point O.

(i) A second block, of mass 3 kg, is attached to the 4 kg block and the system hangs in equilibrium, as shown in the diagram.



Find the extension in the string. *(3 marks)*

(ii) The block of mass 3 kg becomes detached from the 4 kg block and falls to the ground. The 4 kg block now begins to move vertically upwards.

Find the extension of the string when the 4 kg block is next at rest. *(6 marks)*

(iii) Find the extension of the string when the speed of the 4 kg block is a maximum. *(3 marks)*

QUESTION PART REFERENCE

8 Vicky has mass 65 kg and is skydiving. She steps out of a helicopter and falls vertically. She then waits a short period of time before opening her parachute. The parachute opens at time $t = 0$ when her speed is $19.6\,\text{m s}^{-1}$, and she then experiences an air resistance force of magnitude $260v$ newtons, where $v\,\text{m s}^{-1}$ is her speed at time t seconds.

(a) When $t > 0$:

(i) show that the resultant downward force acting on Vicky is

$$65(9.8 - 4v) \text{ newtons} \qquad \textit{(1 mark)}$$

(ii) show that $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -4(v - 2.45)$. *(2 marks)*

(b) By showing that $\displaystyle\int \frac{1}{v - 2.45}\,\mathrm{d}v = -\int 4\,\mathrm{d}t$, find v in terms of t. *(5 marks)*

QUESTION PART REFERENCE

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MM2B

Unit Mechanics 2B

Monday 20 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 In an Olympic diving competition, Kim, who has mass 58 kg, dives from a fixed platform, 10 metres above the surface of the pool. She leaves the platform with a speed of $2\,\text{m}\,\text{s}^{-1}$.

Assume that Kim's weight is the only force that acts on her after she leaves the platform. Kim is to be modelled as a particle which is initially 1 metre above the platform.

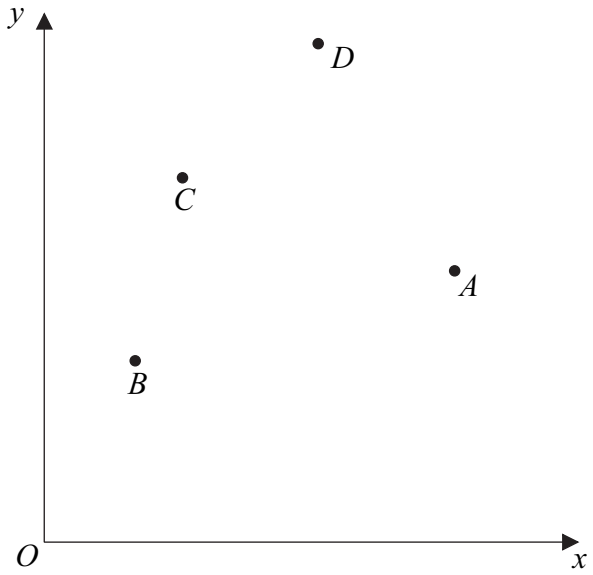
(a) Calculate Kim's initial kinetic energy. *(2 marks)*

(b) By using conservation of energy, find Kim's speed when she is 6 metres below the platform. *(5 marks)*

---

2 The diagram shows four particles, A, B, C and D, which are fixed in a horizontal plane which contains the x- and y-axes, as shown.

Particle A has mass 2 kg and is attached at the point $(9, 6)$.
Particle B has mass 3 kg and is attached at the point $(2, 4)$.
Particle C has mass 8 kg and is attached at the point $(3, 8)$.
Particle D has mass 7 kg and is attached at the point $(6, 11)$.



Find the coordinates of the centre of mass of the four particles. *(5 marks)*

3 A particle moves in a horizontal plane under the action of a single force, **F** newtons. The unit vectors **i** and **j** are directed east and north respectively. At time t seconds, the velocity of the particle, $\mathbf{v}\,\text{m}\,\text{s}^{-1}$, is given by

$$\mathbf{v} = 4\text{e}^{-2t}\,\mathbf{i} + (6t - 3t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(3 marks)*

(b) The mass of the particle is 5 kg.

(i) Find an expression for the force **F** acting on the particle at time t. *(2 marks)*

(ii) Find the magnitude of **F** when $t = 0$. *(2 marks)*

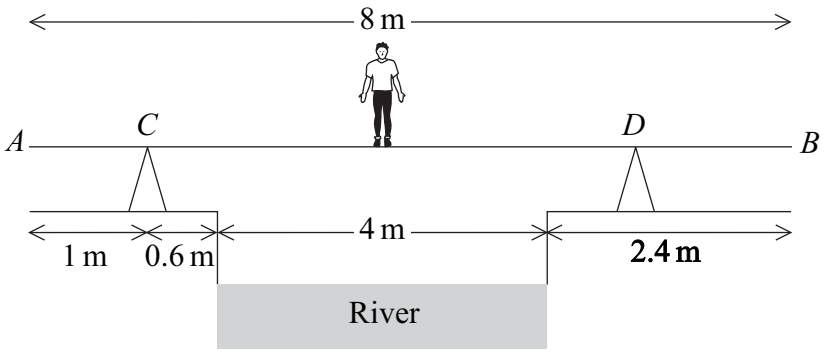
(c) Find the value of t when **F** acts due west. *(2 marks)*

(d) When $t = 0$, the particle is at the point with position vector $(6\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, **r** metres, of the particle at time t. *(5 marks)*

---

4 Ken is trying to cross a river of width 4 m. He has a uniform plank, AB, of length 8 m and mass 17 kg. The ground on both edges of the river bank is horizontal. The plank rests at two points, C and D, on fixed supports which are on opposite sides of the river. The plank is at right angles to both river banks and is horizontal. The distance AC is 1 m, and the point C is at a horizontal distance of 0.6 m from the river bank. Ken, who has mass 65 kg, stands on the plank directly above the middle of the river, as shown in the diagram.



(a) Draw a diagram to show the forces acting on the plank. *(2 marks)*

(b) Given that the reaction on the plank at the point D is $44g$ N, find the horizontal distance of the point D from the nearest river bank. *(4 marks)*

(c) State how you have used the fact that the plank is uniform in your solution. *(1 mark)*

5 A train consists of an engine and five carriages. A constant resistance force of 3000 N acts on the engine, and a constant resistance force of 400 N acts on each of the five carriages.

The maximum speed of the train on a horizontal track is $90\,\text{km}\,\text{h}^{-1}$.

(a) Show that this speed is $25\,\text{m}\,\text{s}^{-1}$. *(1 mark)*

(b) Hence find the maximum power output of the engine. Give your answer in kilowatts. *(3 marks)*

---

6 A car, of mass m kg, is moving along a straight horizontal road. At time t seconds, the car has speed $v\,\text{m}\,\text{s}^{-1}$. As the car moves, it experiences a resistance force of magnitude $2mv^{\frac{5}{4}}$ newtons. No other horizontal force acts on the car.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -2v^{\frac{5}{4}}$$ *(1 mark)*

(b) The initial speed of the car is $16\,\text{m}\,\text{s}^{-1}$.

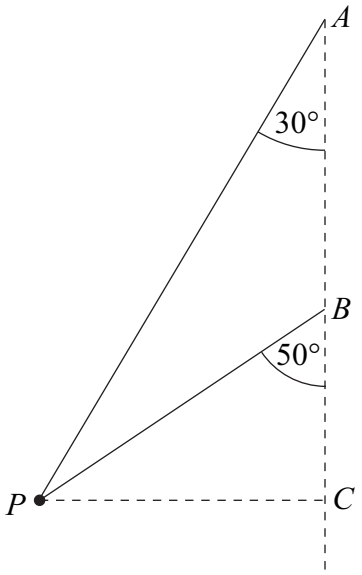
Show that

$$v = \left(\frac{2}{t+1}\right)^4$$ *(5 marks)*

7 Two light inextensible strings each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the fixed points A and B. The point A is vertically above the point B.

The particle moves at a constant speed in a horizontal circle. The centre, C, of this circle is directly below the point B. The two strings are inclined at 30° and 50° to the vertical, as shown in the diagram. Both strings are taut.

As the particle moves in the horizontal circle, the tension in the string BP is 20 N.

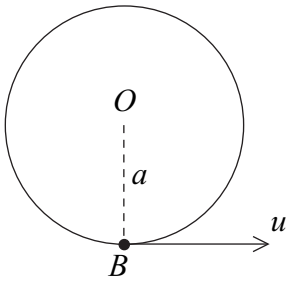


(a) Find the tension in the string AP. *(4 marks)*

(b) The speed of the particle is $5\,\text{m}\,\text{s}^{-1}$.

Find the length of CP, the radius of the horizontal circle. *(4 marks)*

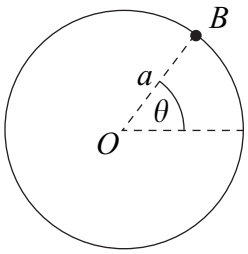
8 A smooth wire is fixed in a vertical plane so that it forms a circle of radius a metres and centre O. A bead, B, of mass 0.3 kg, is threaded on the wire and is set in motion with a speed $u\,\mathrm{m\,s^{-1}}$ at the lowest point of its circular path, as shown in the diagram.



(a) Show that, if the bead is going to make complete revolutions around the wire,

$$u > 2\sqrt{ag}$$ *(3 marks)*

(b) At time t seconds, the angle between OB and the horizontal is θ, as shown in the diagram.



It is given that $u = \sqrt{\frac{9}{2}ag}$.

(i) Find the reaction of the bead on the wire, giving your answer in terms of g and θ. *(5 marks)*

(ii) Find θ when this reaction is zero. *(2 marks)*

9 At a theme park, a light elastic rope is used to bring a carriage to rest at the end of a ride.

The carriage has mass 200 kg and is travelling at $8\,\text{m}\,\text{s}^{-1}$ when the elastic rope is attached to the carriage as it passes over a point O. The other end of the elastic rope is fixed to the point O. The carriage then moves along a horizontal surface until it is brought to rest. The elastic rope is then detached so that the carriage remains at rest.

The elastic rope has natural length 6 m and modulus of elasticity 1800 N. The rope, once taut, remains horizontal throughout the motion.

(a) Calculate the elastic potential energy of the rope when the carriage is 10 m from O. *(3 marks)*

(b) A student's simple model assumes that there are no resistance forces acting on the carriage so that it is brought to rest by the elastic rope alone.

Find the distance of the carriage from O when it is brought to rest. *(3 marks)*

(c) The student improves the model by also including a constant resistance force of 800 N which acts while the carriage is in motion.

Find the distance of the carriage from O when it is brought to rest. *(8 marks)*

END OF QUESTIONS

General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MM2B

Unit Mechanics 2B

Wednesday 25 January 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A plane is dropping packets of aid as it flies over a flooded village. The speed of a packet when it leaves the plane is $60\,\text{m}\,\text{s}^{-1}$. The packet has mass 25 kg.

The packet falls a vertical distance of 34 metres to reach the ground.

(a) Calculate the kinetic energy of the packet when it leaves the plane. *(2 marks)*

(b) Calculate the potential energy lost by the packet as it falls to the ground. *(2 marks)*

(c) Assume that the effect of air resistance on the packet as it falls can be neglected.

(i) Find the kinetic energy of the packet when it reaches the ground. *(2 marks)*

(ii) Hence find the speed of the packet when it reaches the ground. *(2 marks)*

---

2 A particle, of mass 50 kg, moves on a smooth horizontal plane. A single horizontal force

$$[(300t - 60t^2)\,\mathbf{i} + 100\text{e}^{-2t}\,\mathbf{j}] \text{ newtons}$$

acts on the particle at time t seconds.

The vectors **i** and **j** are perpendicular unit vectors.

(a) Find the acceleration of the particle at time t. *(2 marks)*

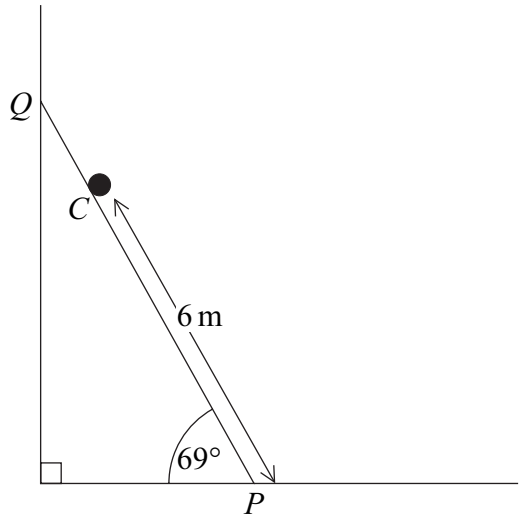
(b) When $t = 0$, the velocity of the particle is $(7\mathbf{i} - 4\mathbf{j})\,\text{m}\,\text{s}^{-1}$.

Find the velocity of the particle at time t. *(4 marks)*

(c) Calculate the speed of the particle when $t = 1$. *(4 marks)*

3 A uniform ladder PQ, of length 8 metres and mass 28 kg, rests in equilibrium with its foot, P, on a rough horizontal floor and its top, Q, leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall and the angle between the ladder and the floor is 69°.

A man, of mass 72 kg, is standing at the point C on the ladder so that the distance PC is 6 metres. The man may be modelled as a particle at C.



(a) Draw a diagram to show the forces acting on the ladder. *(2 marks)*

(b) With the man standing at the point C, the ladder is on the point of slipping.

(i) Show that the magnitude of the reaction between the ladder and the vertical wall is 256 N, correct to three significant figures. *(4 marks)*

(ii) Find the coefficient of friction between the ladder and the horizontal floor. *(4 marks)*

---

4 A car travels along a straight horizontal road. When its speed is $v\,\text{m s}^{-1}$, the car experiences a resistance force of magnitude $25v$ newtons.

(a) The car has a maximum constant speed of $42\,\text{m s}^{-1}$ on this road.

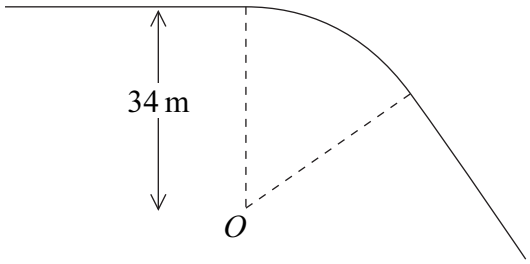
Show that the power being used to propel the car at this speed is 44 100 watts. *(2 marks)*

(b) The car has mass 1500 kg.

Find the acceleration of the car when it is travelling at $15\,\text{m s}^{-1}$ on this road under a power of 44 100 watts. *(4 marks)*

5 A parcel is placed on a flat rough horizontal surface in a van. The van is travelling along a horizontal road. It travels around a bend of radius 34 m at a constant speed. The coefficient of friction between the parcel and the horizontal surface in the van is 0.85 .

Model the parcel as a particle travelling around part of a circle of radius 34 m and centre O, as shown in the diagram.



Find the greatest speed at which the van can travel around the bend without causing the parcel to slide. *(6 marks)*

---

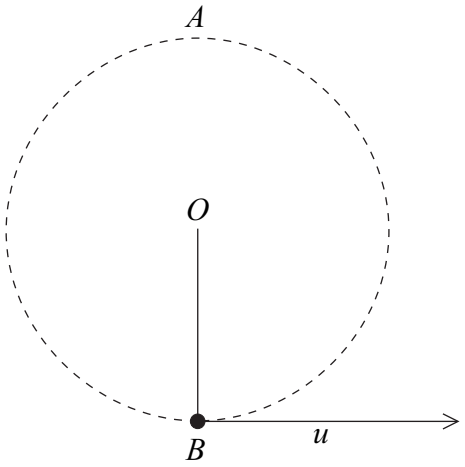
6 Alice places a toy, of mass 0.4 kg, on a slope. The toy is set in motion with an initial velocity of $1\,\text{m}\,\text{s}^{-1}$ down the slope. The resultant force acting on the toy is $(2 - 4v)$ newtons, where $v\,\text{m}\,\text{s}^{-1}$ is the toy's velocity at time t seconds after it is set in motion.

(a) Show that $\dfrac{\text{d}v}{\text{d}t} = -10\,(v - 0.5)$. *(2 marks)*

(b) By using $\displaystyle\int \frac{1}{v - 0.5}\,\text{d}v = -\int 10\,\text{d}t$, find v in terms of t. *(5 marks)*

(c) Find the time taken for the toy's velocity to reduce to $0.55\,\text{m}\,\text{s}^{-1}$. *(3 marks)*

7 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string of length a. With the string taut, the bead is at the point B, vertically below O, when it is set into vertical circular motion with an initial horizontal velocity u, as shown in the diagram.



The string does not become slack in the subsequent motion. The velocity of the bead at the point A, where A is vertically above O, is v.

(a) Show that $v^2 = u^2 - 4ag$. *(2 marks)*

(b) The ratio of the tensions in the string when the bead is at the two points A and B is $2:5$.

(i) Find u in terms of g and a. *(7 marks)*

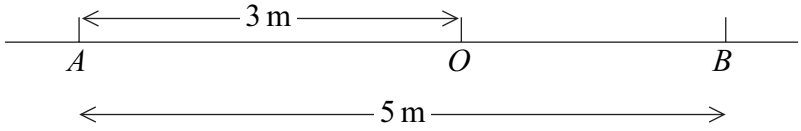
(ii) Find the ratio $u:v$. *(2 marks)*

8 An elastic string has one end attached to a point O fixed on a rough horizontal surface. The other end of the string is attached to a particle of mass 2 kg. The elastic string has natural length 0.8 metres and modulus of elasticity 32 newtons.

The particle is pulled so that it is at the point A, on the surface, 3 metres from the point O.

(a) Calculate the elastic potential energy when the particle is at the point A. *(3 marks)*

(b) The particle is released from rest at the point A and moves in a straight line towards O. The particle is next at rest at the point B. The distance AB is 5 metres.



Find the frictional force acting on the particle as it moves along the surface. *(6 marks)*

(c) Show that the particle does not remain at rest at the point B. *(2 marks)*

(d) The particle next comes to rest at a point C with the string slack.

Find the distance BC. *(2 marks)*

(e) Hence, or otherwise, find the total distance travelled by the particle after it is released from the point A. *(1 mark)*

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MM2B

Unit Mechanics 2B

Thursday 21 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

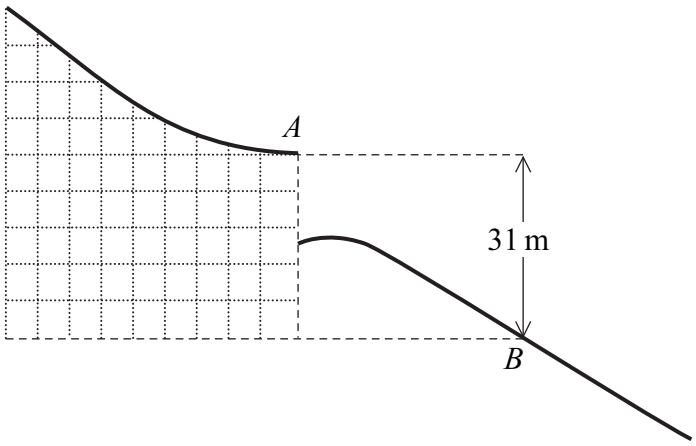
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Alan, of mass 76 kg, performed a ski jump. He took off at the point A at the end of the ski run with a speed of $28\,\text{m}\,\text{s}^{-1}$ and landed at the point B.

The level of the point B is 31 metres vertically below the level of the point A, as shown in the diagram.

Assume that his weight is the only force that acted on Alan during the jump.



(a) Calculate the kinetic energy of Alan when he was at the point A. *(2 marks)*

(b) Calculate the potential energy lost by Alan during the jump as he moved from the point A to the point B. *(2 marks)*

(c) (i) Find the kinetic energy of Alan when he reached the point B. *(2 marks)*

(ii) Hence find the speed of Alan when he reached the point B. *(2 marks)*

---

2 A particle moves in a straight line. At time t seconds, it has velocity $v\,\text{m}\,\text{s}^{-1}$, where

$$v = 6t^2 - 2\text{e}^{-4t} + 8$$

and $t \geqslant 0$.

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) Find the acceleration of the particle when $t = 0.5$. *(2 marks)*

(b) The particle has mass 4 kg.

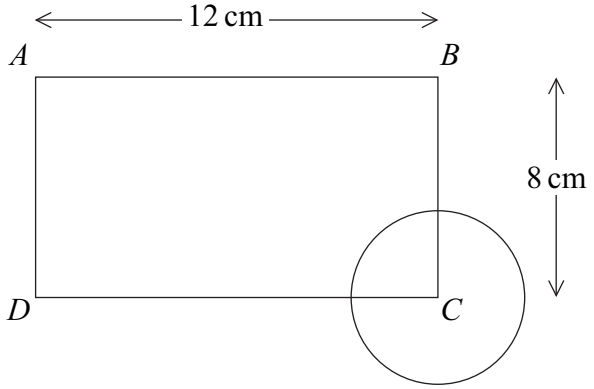
Find the magnitude of the force acting on the particle when $t = 0.5$. *(1 mark)*

(c) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

3 A uniform rectangular lamina $ABCD$, of mass 1.6 kg, has side AB of length 12 cm and side BC of length 8 cm.

To create a logo, a uniform circular lamina, of mass 0.4 kg, is attached. The centre of the circular lamina is at the point C, as shown in the diagram.



(a) Find the distance of the centre of mass of the logo:

(i) from the line AB; *(3 marks)*

(ii) from the line AD. *(3 marks)*

(b) The logo is suspended in equilibrium, with AB horizontal, by two vertical strings. One string is attached at the point A and the other string is attached at the point B.

Find the tension in each of the two strings. *(5 marks)*

---

4 A particle moves on a horizontal plane, in which the unit vectors **i** and **j** are perpendicular.

At time t, the particle's position vector, $\mathbf{r}$, is given by

$$\mathbf{r} = 4\cos 3t\,\mathbf{i} - 4\sin 3t\,\mathbf{j}$$

(a) Prove that the particle is moving on a circle, which has its centre at the origin. *(2 marks)*

(b) Find an expression for the velocity of the particle at time t. *(2 marks)*

(c) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(d) The acceleration of the particle can be written as

$$\mathbf{a} = k\mathbf{r}$$

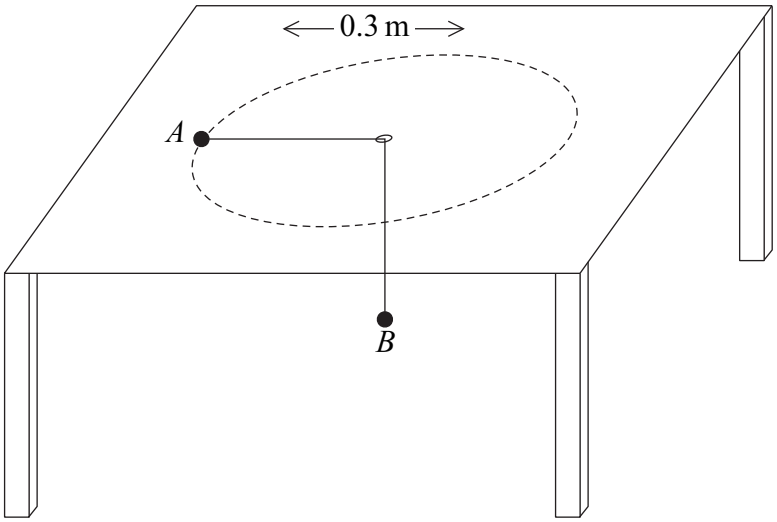
where k is a constant.

Find the value of k. *(2 marks)*

(e) State the direction of the acceleration of the particle. *(1 mark)*

---

5 Two particles, A and B, are connected by a light inextensible string which passes through a hole in a smooth horizontal table. The edges of the hole are also smooth. Particle A, of mass 1.4 kg, moves, on the table, with constant speed in a circle of radius 0.3 m around the hole. Particle B, of mass 2.1 kg, hangs in equilibrium under the table, as shown in the diagram.

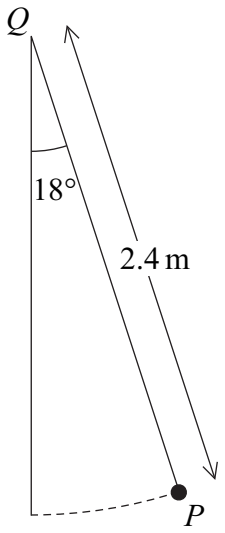


(a) Find the angular speed of particle A. *(4 marks)*

(b) Find the speed of particle A. *(2 marks)*

(c) Find the time taken for particle A to complete one full circle around the hole. *(2 marks)*

6 Simon, a small child of mass 22 kg, is on a swing. He is swinging freely through an angle of 18° on both sides of the vertical. Model Simon as a particle, P, of mass 22 kg, attached to a fixed point, Q, by a light inextensible rope of length 2.4 m.



(a) Find Simon's maximum speed as he swings. *(4 marks)*

(b) Calculate the tension in the rope when Simon's speed is a maximum. *(3 marks)*

---

7 A stone, of mass 5 kg, is projected vertically downwards, in a viscous liquid, with an initial speed of $7\,\text{m}\,\text{s}^{-1}$.

At time t seconds after it is projected, the stone has speed $v\,\text{m}\,\text{s}^{-1}$ and it experiences a resistance force of magnitude $9.8v$ newtons.

(a) When $t \geqslant 0$, show that

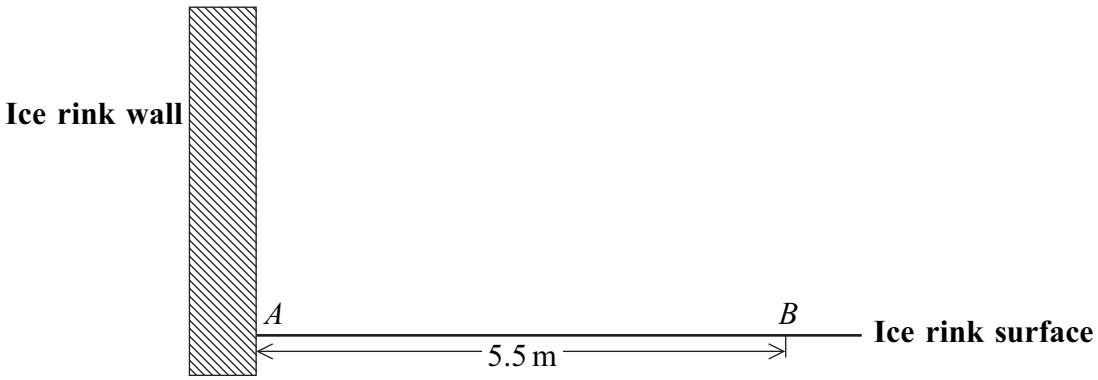
$$\frac{\mathrm{d}v}{\mathrm{d}t} = -1.96(v - 5)$$ *(2 marks)*

(b) Find v in terms of t. *(5 marks)*

8 Zoë carries out an experiment with a block, which she places on the horizontal surface of an ice rink. She attaches one end of a light elastic string to a fixed point, A, on a vertical wall at the edge of the ice rink at the height of the surface of the ice rink.

The block, of mass 0.4 kg, is attached to the other end of the string. The string has natural length 5 m and modulus of elasticity 120 N.

The block is modelled as a particle which is placed on the surface of the ice rink at a point B, where AB is perpendicular to the wall and of length 5.5 m.



The block is set into motion at the point B with speed $9\,\text{m}\,\text{s}^{-1}$ directly towards the point A. The string remains horizontal throughout the motion.

(a) Initially, Zoë assumes that the surface of the ice rink is smooth.

Using this assumption, find the speed of the block when it reaches the point A.

(4 marks)

(b) Zoë now assumes that friction acts on the block. The coefficient of friction between the block and the surface of the ice rink is μ.

(i) Find, in terms of g and μ, the speed of the block when it reaches the point A.

(6 marks)

(ii) The block rebounds from the wall in the direction of the point B. The speed of the block immediately after the rebound is half of the speed with which it hit the wall.

Find μ if the block comes to rest just as it reaches the point B. *(6 marks)*

General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MM2B

Unit Mechanics 2B

Monday 28 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Tim is playing cricket. He hits a ball at a point A. The speed of the ball immediately after being hit is $11\,\text{m s}^{-1}$.

The ball strikes a tree at a point B. The height of B is 5 metres above the height of A.

The ball is to be modelled as a particle of mass 0.16 kg being acted upon only by gravity.

(a) Calculate the initial kinetic energy of the ball. *(2 marks)*

(b) Calculate the potential energy gained by the ball as it moves from the point A to the point B. *(2 marks)*

(c) (i) Find the kinetic energy of the ball immediately before it strikes the tree. *(2 marks)*

(ii) Hence find the speed of the ball immediately before it strikes the tree. *(2 marks)*

---

2 A particle moves in a horizontal plane. The vectors **i** and **j** are perpendicular unit vectors in the horizontal plane. At time t seconds, the velocity of the particle, $\mathbf{v}\,\text{m s}^{-1}$, is given by

$$\mathbf{v} = 12\cos\left(\frac{\pi}{3}t\right)\mathbf{i} - 9t^2\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(b) The particle, which has mass 4 kg, moves under the action of a single force, **F** newtons.

(i) Find an expression for the force **F** in terms of t. *(2 marks)*

(ii) Find the magnitude of **F** when $t = 3$. *(2 marks)*

(c) When $t = 3$, the particle is at the point with position vector $(4\mathbf{i} - 2\mathbf{j})$ m.

Find the position vector, **r** metres, of the particle at time t. *(5 marks)*

---

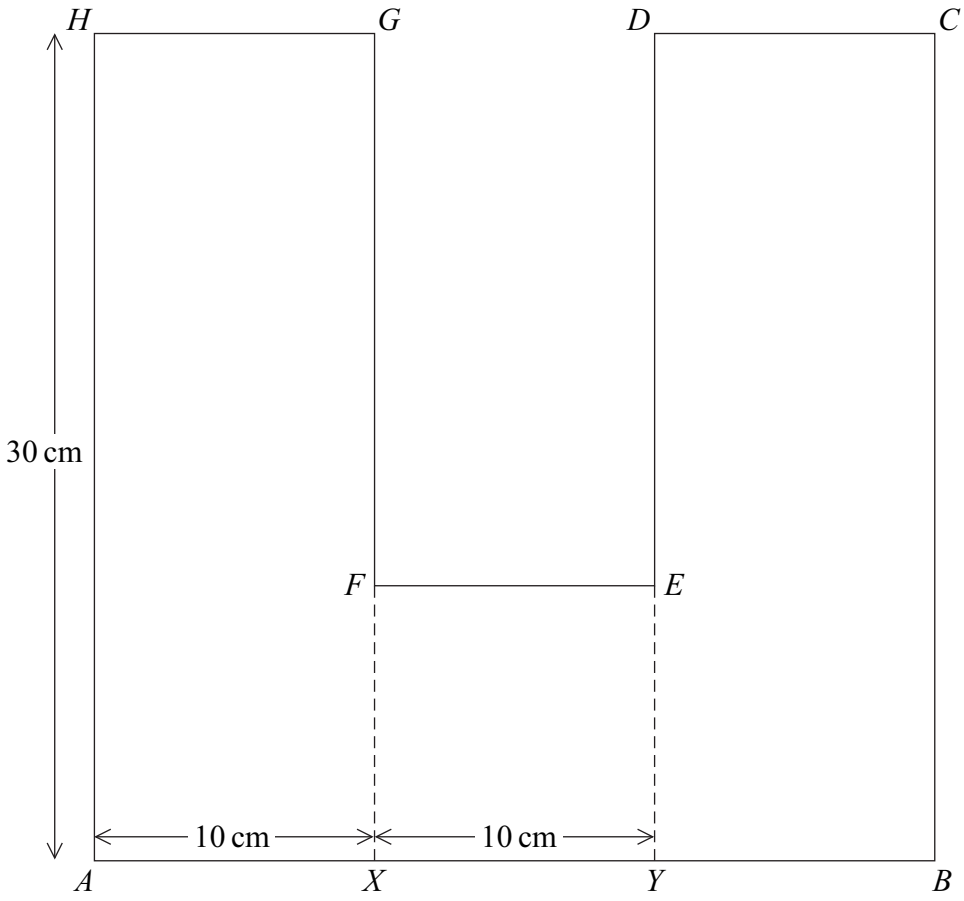
3 A van, of mass 1500 kg, travels at a constant speed of $22\,\text{m s}^{-1}$ up a slope inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{25}$.

The van experiences a resistance force of 8 000 N.

Find the power output of the van's engine, giving your answer in kilowatts. *(5 marks)*

4 The diagram shows a uniform lamina which is in the shape of two identical rectangles *AXGH* and *YBCD* and a square *XYEF*, arranged as shown.

The length of *AX* is 10 cm, the length of *XY* is 10 cm and the length of *AH* is 30 cm.



(a) Explain why the centre of mass of the lamina is 15 cm from *AH*. *(1 mark)*

(b) Find the distance of the centre of mass of the lamina from *AB*. *(3 marks)*

(c) The lamina is freely suspended from the point *H*.

Find, to the nearest degree, the angle between *HG* and the horizontal when the lamina is in equilibrium. *(4 marks)*

5 A particle, of mass 12 kg, is moving along a straight horizontal line. At time t seconds, the particle has speed $v\,\mathrm{m\,s^{-1}}$. As the particle moves, it experiences a resistance force of magnitude $4v^{\frac{1}{3}}$. No other horizontal force acts on the particle.

The initial speed of the particle is $8\,\mathrm{m\,s^{-1}}$.

(a) Show that

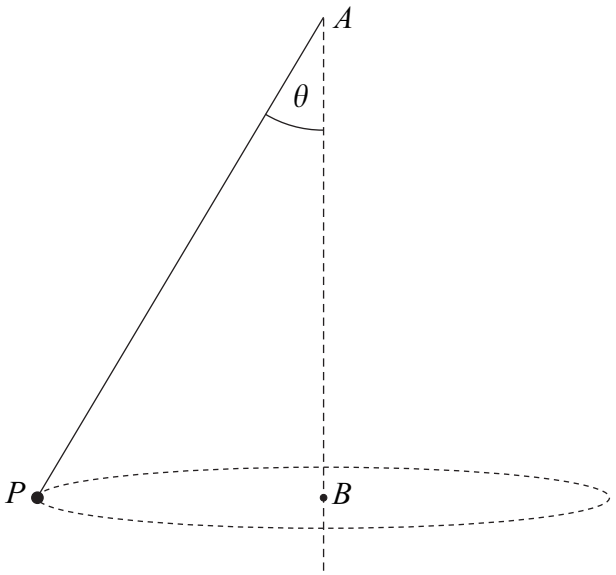
$$v = \left(4 - \frac{2}{9}t\right)^{\frac{3}{2}}$$ *(6 marks)*

(b) Find the value of t when the particle comes to rest. *(1 mark)*

---

6 A light inextensible string has one end attached to a particle, P, of mass 2 kg. The other end of the string is attached to the fixed point A. The point A is vertically above the point B. The particle moves at a constant speed in a horizontal circle of radius 0.8 m and centre B. The tension in the string is 34 N.

The string is inclined at an angle θ to the vertical, as shown in the diagram.



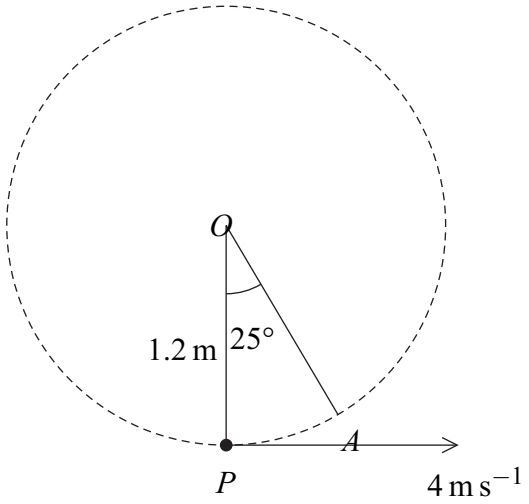
(a) Find the angle θ. *(3 marks)*

(b) Find the speed of the particle. *(3 marks)*

(c) Find the time taken for the particle to make one complete revolution. *(2 marks)*

7 A small ball, of mass 3 kg, is suspended from a fixed point O by a light inextensible string of length 1.2 m. Initially, the string is taut and the ball is at the point P, vertically below O. The ball is then set into motion with an initial horizontal velocity of $4\,\text{m s}^{-1}$.

The ball moves in a vertical circle, centre O. The point A, on the circle, is such that angle AOP is 25°, as shown in the diagram.



(a) Find the speed of the ball at the point A. *(4 marks)*

(b) Find the tension in the string when the ball is at the point A. *(3 marks)*

---

8 (a) An elastic string has natural length l and modulus of elasticity λ. The string is stretched from length l to length $l + e$.

Show, by integration, that the work done in stretching the string is $\frac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string. The other end of the string is attached to a fixed point O.

The string has natural length 1.6 m and modulus of elasticity 392 N.

(i) Find the extension of the string when the particle hangs in equilibrium. *(2 marks)*

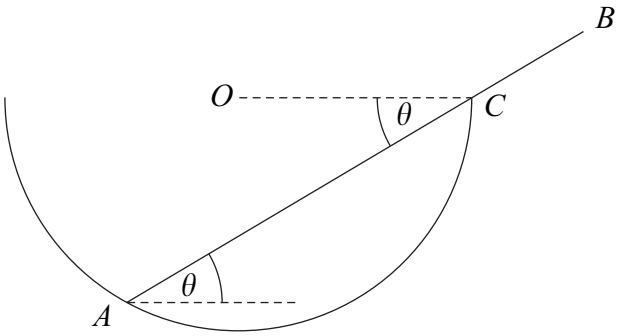
(ii) The particle is pulled down to a point A, which is 2.2 m below the point O.

Calculate the elastic potential energy in the string. *(3 marks)*

(iii) The particle is released when it is at rest at the point A.

Calculate the distance of the particle from the point A when its speed first reaches $0.8\,\text{m s}^{-1}$. *(5 marks)*

9 A smooth hollow hemisphere, of radius a and centre O, is fixed so that its rim is in a horizontal plane. A smooth uniform rod AB, of mass m, is in equilibrium, with one end A resting on the inside of the hemisphere and the point C on the rod being in contact with the rim of the hemisphere. The rod, of length l, is inclined at an angle θ to the horizontal, as shown in the diagram.



(a) Explain why the reaction between the rod and the hemisphere at point A acts through O. *(1 mark)*

(b) Draw a diagram to show the forces acting on the rod. *(2 marks)*

(c) Show that $l = \dfrac{4a\cos 2\theta}{\cos\theta}$. *(5 marks)*

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MM2B

Unit Mechanics 2B

Thursday 13 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A particle, of mass 3 kg, moves along a straight line. At time t seconds, the displacement, s metres, of the particle from the origin is given by

$$s = 8t^3 + 15$$

(a) Find the velocity of the particle at time t. *(2 marks)*

(b) Find the magnitude of the resultant force acting on the particle when $t = 2$. *(4 marks)*

---

2 Carol, a circus performer, is on a swing. She jumps off the swing and lands in a safety net. When Carol leaves the swing, she has a speed of $7\,\text{m}\,\text{s}^{-1}$ and she is at a height of 8 metres above the safety net.

Carol is to be modelled as a particle of mass 52 kg being acted upon only by gravity.

(a) Find the kinetic energy of Carol when she leaves the swing. *(2 marks)*

(b) Show that the kinetic energy of Carol when she hits the net is 5350 J, correct to three significant figures. *(3 marks)*

(c) Find the speed of Carol when she hits the net. *(3 marks)*

---

3 A particle, of mass 10 kg, moves on a smooth horizontal plane. At time t seconds, the acceleration of the particle is given by

$$\{(40t + 3t^2)\,\mathbf{i} + 20\text{e}^{-4t}\,\mathbf{j}\}\,\text{m}\,\text{s}^{-2}$$

where the vectors **i** and **j** are perpendicular unit vectors.

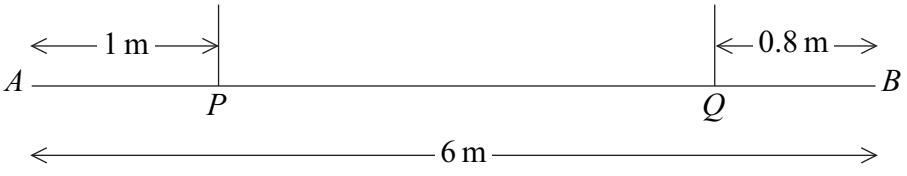
(a) At time $t = 1$, the velocity of the particle is $(6\mathbf{i} - 5\text{e}^{-4}\mathbf{j})\,\text{m}\,\text{s}^{-1}$.

Find the velocity of the particle at time t. *(5 marks)*

(b) Calculate the initial speed of the particle. *(3 marks)*

---

4 A uniform plank AB, of length 6 m, has mass 25 kg. It is supported in equilibrium in a horizontal position by two vertical inextensible ropes. One of the ropes is attached to the plank at the point P and the other rope is attached to the plank at the point Q, where $AP = 1\,\text{m}$ and $QB = 0.8\,\text{m}$, as shown in the diagram.



(a) (i) Find the tension in each rope. *(5 marks)*

(ii) State how you have used the fact that the plank is uniform in your solution. *(1 mark)*

(b) A particle of mass m kg is attached to the plank at point B, and the tension in each rope is now the same.

Find m. *(6 marks)*

---

5 Tom is travelling on a train which is moving at a constant speed of $15\,\text{m s}^{-1}$ on a horizontal track. Tom has placed his mobile phone on a rough horizontal table. The coefficient of friction between the phone and the table is 0.2.

The train moves round a bend of constant radius. The phone does not slide as the train travels round the bend.

Model the phone as a particle moving round part of a circle, with centre O and radius r metres.

Find the least possible value of r. *(4 marks)*

---

6 A car accelerates from rest along a straight horizontal road.

The car's engine produces a constant horizontal force of magnitude 4000 N.

At time t seconds, the speed of the car is $v\,\text{m s}^{-1}$, and a resistance force of magnitude $40v$ newtons acts upon the car.

The mass of the car is 1600 kg.

(a) Show that $\dfrac{\text{d}v}{\text{d}t} = \dfrac{100 - v}{40}$. *(2 marks)*

(b) Find the velocity of the car at time t. *(6 marks)*

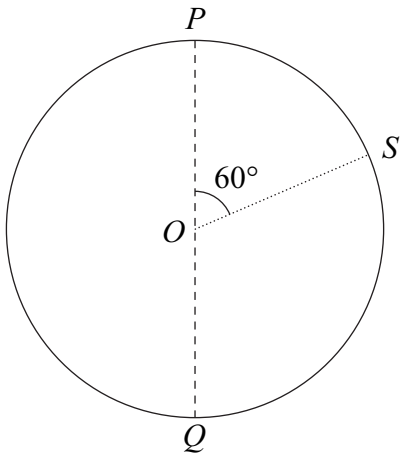
7 A train, of mass 22 tonnes, moves along a straight horizontal track. A constant resistance force of 5000 N acts on the train. The power output of the engine of the train is 240 kW.

Find the acceleration of the train when its speed is $20\,\text{m s}^{-1}$. *(6 marks)*

8 A bead, of mass m, moves on a smooth circular ring, of radius a and centre O, which is fixed in a vertical plane. At P, the highest point on the ring, the speed of the bead is $2u$; at Q, the lowest point on the ring, the speed of the bead is $5u$.

(a) Show that $u = \sqrt{\frac{4ag}{21}}$. *(4 marks)*

(b) S is a point on the ring so that angle POS is 60°, as shown in the diagram.



Find, in terms of m and g, the magnitude of the reaction of the ring on the bead when the bead is at S. *(5 marks)*

---

9 Two particles, A and B, are connected by a light elastic string that passes through a hole at a point O in a rough horizontal table. The edges of the hole are smooth. Particle A has a mass of 8 kg and particle B has a mass of 3 kg.

The elastic string has natural length 3 metres and modulus of elasticity 60 newtons.

Initially, particle A is held 3.5 metres from the point O on the surface of the table and particle B is held at a point 2 metres vertically below O.

The coefficient of friction between the table and particle A is 0.4.

The two particles are released from rest.

(a) (i) Show that initially particle A moves towards the hole in the table. *(3 marks)*

(ii) Show that initially particle B also moves towards the hole in the table. *(2 marks)*

(b) Calculate the **initial** elastic potential energy in the string. *(2 marks)*

(c) Particle A comes permanently to rest when it has moved 0.46 metres, at which time particle B is still moving upwards.

Calculate the distance that particle B has moved when it is at rest for the first time. *(7 marks)*

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MM2B

Unit Mechanics 2B

Monday 23 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 An eagle has caught a salmon of mass $3\,\text{kg}$ to take to its nest. When the eagle is flying with speed $8\,\text{m s}^{-1}$, it drops the salmon. The salmon falls a vertical distance of 13 metres back into the sea.

The salmon is to be modelled as a particle. The salmon's weight is the only force that acts on it as it falls to the sea.

(a) Calculate the kinetic energy of the salmon when it is dropped by the eagle.

[2 marks]

(b) Calculate the potential energy lost by the salmon as it falls to the sea.

[2 marks]

(c) (i) Find the kinetic energy of the salmon when it reaches the sea.

[2 marks]

(ii) Hence find the speed of the salmon when it reaches the sea.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

2 A particle has mass $6\,\text{kg}$. A single force $(24\text{e}^{-2t}\mathbf{i} - 12t^3\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t.

[2 marks]

(b) At time $t = 0$, the velocity of the particle is $(-7\mathbf{i} - 4\mathbf{j})\,\text{m s}^{-1}$.

Find the velocity of the particle at time t.

[4 marks]

(c) Find the speed of the particle when $t = 0.5$.

[4 marks]

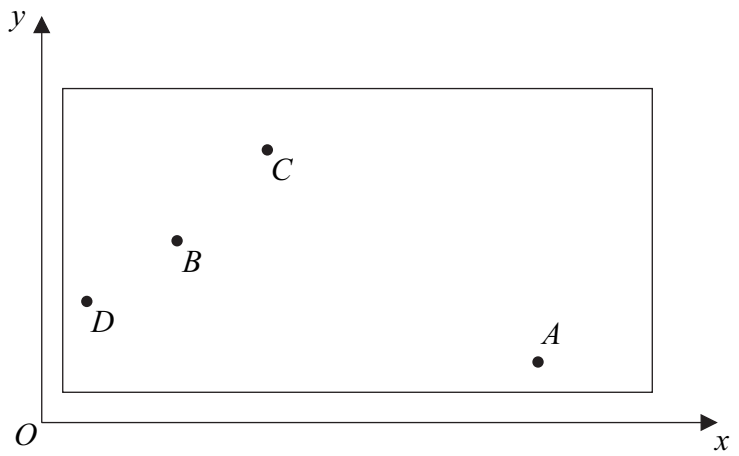
QUESTION PART REFERENCE

Answer space for question 2

3 Four tools are attached to a board.

The board is to be modelled as a uniform lamina and the four tools as four particles.

The diagram shows the lamina, the four particles A, B, C and D, and the x and y axes.



The lamina has mass $5\,\text{kg}$ and its centre of mass is at the point $(7, 6)$.

Particle A has mass $4\,\text{kg}$ and is at the point $(11, 2)$.
Particle B has mass $3\,\text{kg}$ and is at the point $(3, 6)$.
Particle C has mass $7\,\text{kg}$ and is at the point $(5, 9)$.
Particle D has mass $1\,\text{kg}$ and is at the point $(1, 4)$.

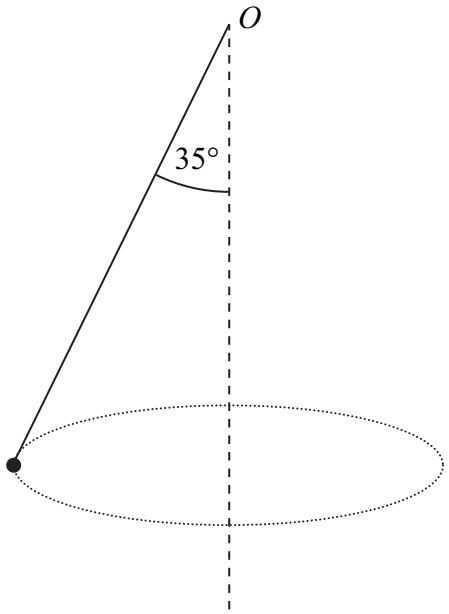
Find the coordinates of the centre of mass of the system of board and tools.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 3

4 A particle, of mass $0.8\,\text{kg}$, is attached to one end of a light inextensible string. The other end of the string is attached to the fixed point O. The particle is set in motion, so that it moves in a horizontal circle at constant speed, with the string at an angle of $35°$ to the vertical. The centre of this circle is vertically below O, as shown in the diagram.



The particle moves in a horizontal circle and completes 20 revolutions each minute.

(a) Find the angular speed of the particle in radians per second.

[2 marks]

(b) Find the tension in the string.

[3 marks]

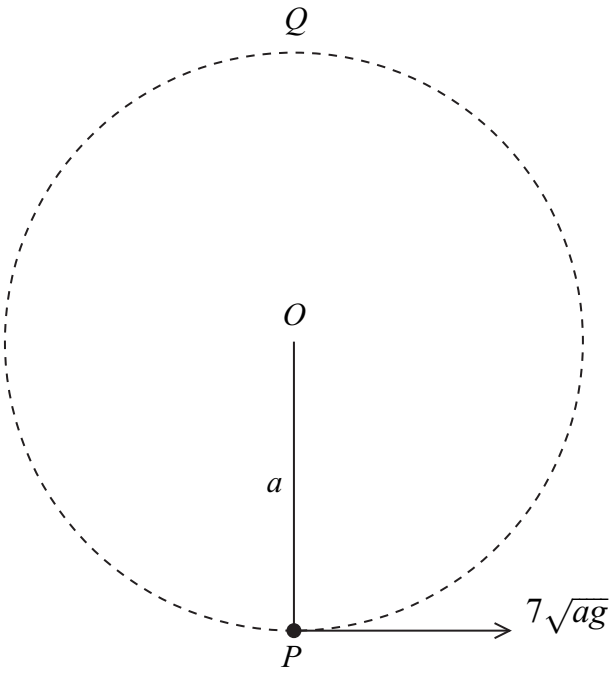
(c) Find the radius of the horizontal circle.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 4

5 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end of the string. The particle is moving in a vertical circle with centre O. The point Q is the highest point of the particle's path. When the particle is at P, vertically below O, the string is taut and the particle is moving with speed $7\sqrt{ag}$, as shown in the diagram.



(a) Find, in terms of g and a, the speed of the particle at the point Q.

[4 marks]

(b) Find, in terms of g and m, the tension in the string when the particle is at Q.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

6 A puck, of mass $m\,\text{kg}$, is moving in a straight line across smooth horizontal ice. At time t seconds, the puck has speed $v\,\text{m s}^{-1}$. As the puck moves, it experiences an air resistance force of magnitude $0.3mv^{\frac{1}{3}}$ newtons, until it comes to rest. No other horizontal forces act on the puck.

When $t = 0$, the speed of the puck is $8\,\text{m s}^{-1}$.

Model the puck as a particle.

(a) Show that

$$v = (4 - 0.2t)^{\frac{3}{2}}$$

[6 marks]

(b) Find the value of t when the puck comes to rest.

[2 marks]

(c) Find the distance travelled by the puck as its speed decreases from $8\,\text{m s}^{-1}$ to zero.

[5 marks]

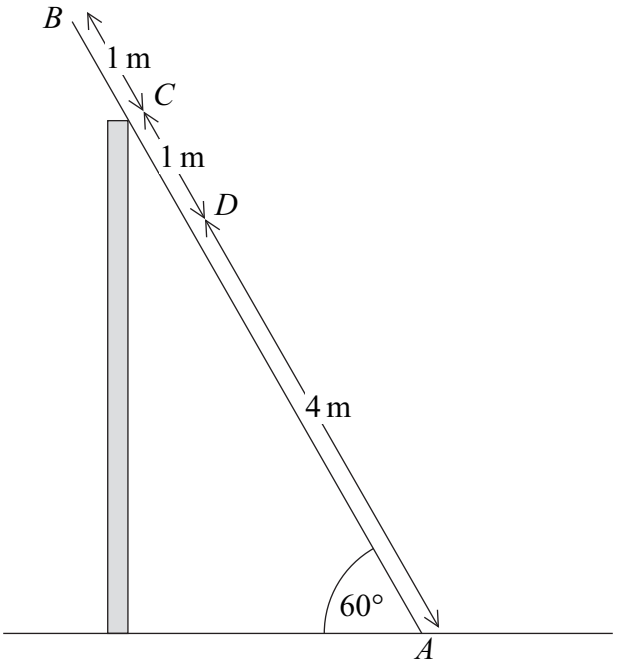
QUESTION PART REFERENCE

Answer space for question 6

7 A uniform ladder AB, of length 6 metres and mass $22\,\text{kg}$, rests with its foot, A, on rough horizontal ground. The ladder rests against the top of a smooth vertical wall at the point C, where the length AC is 5 metres. The vertical plane containing the ladder is perpendicular to the wall, and the angle between the ladder and the ground is $60°$. A man, of mass $88\,\text{kg}$, is standing on the ladder.

The man may be modelled as a particle at the point D, where the length of AD is 4 metres.

The ladder is on the point of slipping.



(a) Draw a diagram to show the forces acting on the ladder.

[2 marks]

(b) Find the coefficient of friction between the ladder and the horizontal ground.

[6 marks]

QUESTION PART REFERENCE

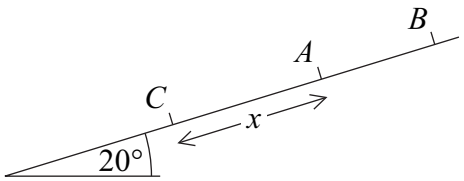
Answer space for question 7

Do not write outside the box

8 An elastic string has natural length 1.5 metres and modulus of elasticity 120 newtons. One end of the string is attached to a fixed point, A, on a rough plane inclined at $20°$ to the horizontal. The other end of the elastic string is attached to a particle of mass $4\,\text{kg}$. The coefficient of friction between the particle and the plane is 0.8.

The three points, A, B and C, lie on a line of greatest slope.

The point C is x metres from A, as shown in the diagram. The particle is released from rest at C and moves up the plane.



(a) Show that, as the particle moves up the plane, the frictional force acting on the particle is $29.5\,\text{N}$, correct to three significant figures.

[3 marks]

(b) The particle comes to rest for an instant at B, which is 2 metres from A.

The particle then starts to move back towards A.

(i) Find x.

[8 marks]

(ii) Find the acceleration of the particle as it starts to move back towards A.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MM2B

Unit Mechanics 2B

Monday 22 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A particle, of mass $4\,\text{kg}$, moves in a horizontal plane under the action of a single force, $\mathbf{F}$ newtons. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in the horizontal plane, perpendicular to each other.

At time t seconds, the velocity of the particle, $\mathbf{v}\,\text{m s}^{-1}$, is given by

$$\mathbf{v} = 4\cos 2t\ \mathbf{i} + 3\sin t\ \mathbf{j}$$

(a) (i) Find an expression for the force, $\mathbf{F}$, acting on the particle at time t seconds.

[3 marks]

(ii) Find the magnitude of $\mathbf{F}$ when $t = \pi$.

[2 marks]

(b) When $t = 0$, the particle is at the point with position vector $(2\mathbf{i} - 14\mathbf{j})$ metres. Find the position vector, $\mathbf{r}$ metres, of the particle at time t seconds.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

2 A uniform rod AB, of mass $4\,\text{kg}$ and length 6 metres, has three masses attached to it. A $3\,\text{kg}$ mass is attached at the end A and a $5\,\text{kg}$ mass is attached at the end B. An $8\,\text{kg}$ mass is attached at a point C on the rod.

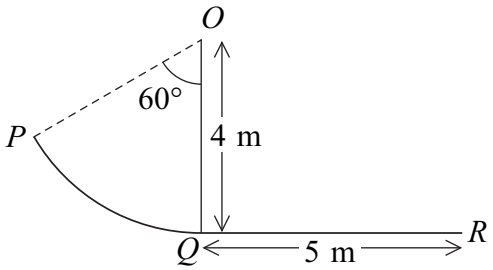
Find the distance AC if the centre of mass of the system is $4.3\,\text{m}$ from point A.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

3 A diagram shows a children's slide, PQR.



Simon, a child of mass $32\,\text{kg}$, uses the slide, starting from rest at P. The curved section of the slide, PQ, is one sixth of a circle of radius 4 metres so that the child is travelling horizontally at point Q. The centre of this circle is at point O, which is vertically above point Q. The section QR is horizontal and of length 5 metres.

Assume that air resistance may be ignored.

(a) Assume that the two sections of the slide, PQ and QR, are both smooth.

(i) Find the kinetic energy of Simon when he reaches the point R.

[3 marks]

(ii) Hence find the speed of Simon when he reaches the point R.

[2 marks]

(b) In fact, the section QR is rough.

Assume that the section PQ is smooth.

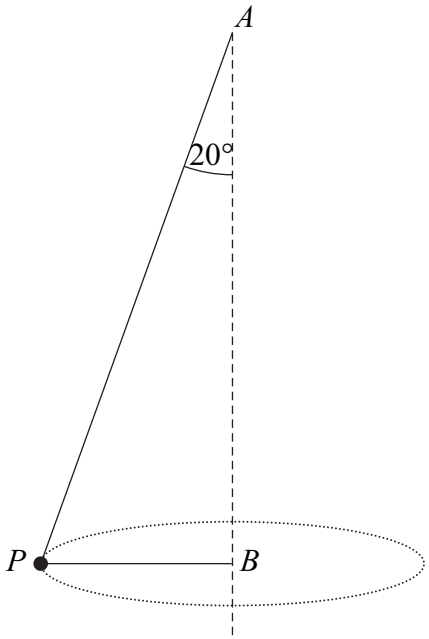
Find the coefficient of friction between Simon and the section QR if Simon comes to rest at the point R.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

4 A particle, P, of mass $5\,\text{kg}$ is attached to two light inextensible strings, AP and BP. The other ends of the strings are attached to the fixed points A and B. The point A is vertically above the point B. The particle moves at a constant speed, $v\,\text{m s}^{-1}$, in a horizontal circle of radius 0.6 metres with centre B. The string AP is inclined at $20°$ to the vertical, as shown in the diagram. Both strings are taut when the particle is moving.



(a) Find the tension in the string AP.

[3 marks]

(b) The speed of the particle is $v\,\text{m s}^{-1}$.

Show that the tension, T_{BP}, in the string BP is given by

$$T_{BP} = \frac{25}{3}v^2 - 5g\tan 20°$$

[3 marks]

(c) Find v when the tensions in the two strings are equal.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

5 An item of clothing is placed inside a washing machine. The drum of the washing machine has radius $30\,\text{cm}$ and rotates, about a fixed horizontal axis, at a constant angular speed of 900 revolutions per minute.

Model the item of clothing as a particle of mass $0.8\,\text{kg}$ and assume that the clothing travels in a vertical circle with constant angular speed.

Find the minimum magnitude of the normal reaction force exerted by the drum on the clothing and find the maximum magnitude of the normal reaction force exerted by the drum on the clothing.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 5

6 A van, of mass $1400\,\text{kg}$, is accelerating at a constant rate of $0.2\,\text{m s}^{-2}$ as it travels up a slope inclined at an angle θ to the horizontal.

The van experiences total resistance forces of $4000\ \text{N}$.

When the van is travelling at a speed of $20\,\text{m s}^{-1}$, the power output of the van's engine is $91.1\,\text{kW}$.

Find θ.

[9 marks]

QUESTION PART REFERENCE

Answer space for question 6

7 A parachutist, of mass $72\,\text{kg}$, is falling vertically. He opens his parachute at time $t = 0$ when his speed is $30\,\text{m s}^{-1}$. He then experiences an air resistance force of magnitude $240v$ newtons, where $v\,\text{m s}^{-1}$ is his speed at time t seconds.

(a) When $t > 0$, show that $-\frac{3}{10}\frac{\text{d}v}{\text{d}t} = v - 2.94$.

[2 marks]

(b) Find v in terms of t.

[5 marks]

(c) Sketch a graph to show how, for $t \geqslant 0$, the parachutist's speed varies with time.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

8 Carol, a bungee jumper of mass $70\,\text{kg}$, is attached to one end of a light elastic cord of natural length 26 metres and modulus of elasticity $1456\ \text{N}$. The other end of the cord is attached to a fixed horizontal platform which is at a height of 69 metres above the ground.

Carol steps off the platform at the point where the cord is attached and falls vertically. Hooke's law can be assumed to apply whilst the cord is taut.

Model Carol as a particle and assume air resistance to be negligible.

When Carol has fallen x m, her speed is $v\,\text{m s}^{-1}$.

(a) By considering energy, show that

$$5v^2 = 306x - 4x^2 - 2704 \quad \text{for} \quad x \geqslant 26$$

[4 marks]

(b) Why is the expression found in part **(a)** not true when x takes values less than 26?

[1 mark]

(c) Find the maximum value of x.

[2 marks]

(d) (i) Find the distance fallen by Carol when her speed is a maximum.

[2 marks]

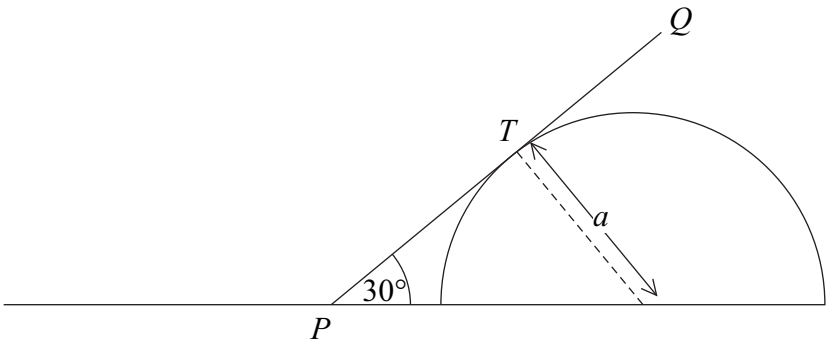
(ii) Hence find Carol's maximum speed.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 8

9 A uniform rod, PQ, of length $2a$, rests with one end, P, on rough horizontal ground and a point T resting on a rough fixed prism of semicircular cross-section of radius a, as shown in the diagram. The rod is in a vertical plane which is parallel to the prism's cross-section. The coefficient of friction at both P and T is μ.



The rod is on the point of slipping when it is inclined at an angle of $30°$ to the horizontal.

Find the value of μ.

[8 marks]

QUESTION PART REFERENCE

Answer space for question 9